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Near-rectilinear quasi-periodic trajectories in the three-body problem

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SUMMARY

We deal with the analysis of the spatial three-body problem in the various regimes where the Hamiltonian is split as the sum of two Keplerian systems plus a small perturbation. This is a region of the phase space $T^*\mathbb{R}^6$ where the perturbation is small (cf. [2]), the so called perturbing region. In particular we prove the existence of quasi-periodic motions where the inner particles describe bounded near-rectilinear trajectories whereas the outer particle follows an orbit lying near the invariable plane. These motions fill in five-dimensional invariant tori. Moreover, the inner particles move in orbits either near an axis perpendicular to the invariable plane or near the invariable plane in almost straight motions (cf. [3]).

By averaging over the mean anomalies, truncating higher-order terms and using singular reduction theory we get a one-degree-of-freedom Hamiltonian system defined in a singular reduced space. In (cf. [2]) we analyse the relative equilibria and bifurcations. Three of the relative equilibria are elliptic points that correspond to near-rectilinear motions of the inner bodies and these are the ones we consider here. The reconstruction of the KAM 5-tori surrounding the three equilibria is carried out. By means of our reduction process we regularise the double inner collisions and this allows us to build sets of action-angle coordinates needed to apply KAM theory. In order to achieve the existence of the quasi-periodic motions we use a theorem by Han, Li and Yi (cf. [1]) that allows us to handle the high-order degeneracy of the Hamiltonians involved in the process.

This work is part of the second author's PhD thesis (cf. [4]).

Keywords: Spatial three-body problem, symmetry, Deprit's coordinates for the N-body problem, regular and singular reductions, regularisation of the double inner collisions, quasi-periodic motions of rectilinear type, KAM theory for properly degenerate Hamiltonians

AMS Classification: 70F07, 70K65, 37J15, 37J40, 37J20

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