

Valuative Criterion, Arcs and Adjacencies of Plane Curves

Pe Pereira, M.¹, Fernández de Bobadilla, J., Popescu Pampu, P.

SUMMARY

We study two notions of adjacency between prime divisors lying above the origin O of the complex plane. These divisors are the exceptional components of a composition of blow ups over the origin of \mathbb{C}^2 . We say that E is adjacent to F if there is a deformation of germs of curves at O whose special and general members have strict transforms on a common model of E and F which intersect F and E respectively at smooth points of the exceptional divisor. We say that E is Nash-adjacent to F if the closure of the space of arcs associated to F is contained in the closure of the one associated to E . Nash-adjacency implies adjacency. Our first main theorem states that adjacency is equivalent to the purely combinatorial fact that the divisorial valuation associated to E is inferior to the one associated to F . This is very much related to a previous work of M. Alberich and J. Roe. We see that this condition is equivalent to finitely many inequalities which allow us to give an algorithm to find all the adjacent divisors to a given one. Some of these results extend to non-prime divisors. A similar characterisation is true for adjacencies between non-prime divisors. Our second main theorem states that Nash-adjacency is also combinatorial

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¹ICMAT

email: maria.pe@icmat.es