

A new two-parameter probability distribution with bounded support

P. Jodrá¹, M. D. Jiménez-Gamero²

SUMMARY

A probability distribution with bounded support is derived from the exponential-geometric distribution given in [1]. To be more precise, the cumulative distribution function of the new law is the following

$$F(x; \alpha, \beta) = \frac{(1 + \beta)x^\alpha}{1 + \beta x^\alpha}, \quad 0 < x < 1, \quad \alpha > 0, \quad \beta > -1,$$

where α and β are shape parameters. Some statistical properties are studied. Specifically, closed-form expressions are provided for the non-central moments, quantile function, moments of the order statistics and the Shannon entropy, among others. Additionally, we show that the new probability distribution has increasing generalized failure rate.

The parameter estimation problem is carried out by the maximum likelihood (ML) method and its performance is assessed by means of a Monte Carlo simulation study. The Fisher information matrix and the asymptotic distribution of the ML estimators are also provided. Unified proofs can be given based on the results in [3]. A real data set provided in [2] is used to exemplify that the new model may provide a better fit than other probability distributions with bounded domain.

Finally, it is introduced a regression model based on the assumptions that the response variable has distribution F and is related to a set of covariates through a regression structure. A real data application illustrates the suitability of the proposed regression model.

Keywords: Exponential-geometric distribution, bounded domain, data analysis.

AMS Classification: 60E05, 62F10

References

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¹Dpto. de Métodos Estadísticos, Universidad de Zaragoza
C/ María de Luna 3, 50018 Zaragoza, Spain
E-mail: pjodra@unizar.es

²Dpto. de Estadística e Investigación Operativa, Universidad de Sevilla
Avd. Reina Mercedes s.n, 41012 Sevilla, Spain
E-mail: dolores@us.es