Fourteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 12–15th 2016

On the stochastic comparison of two models of imperfect repairs for a gamma deteriorating system

Sophie MERCIER[1], **Inmaculada T. CASTRO**[2]

SUMMARY

A system is considered, which is subject to a non-decreasing accumulating random degradation, classically modeled by a non homogeneous gamma process [2]. When the degradation level reaches a certain level, the system is considered to be no longer able to function satisfactorily. Hence the need for maintenance actions, whose aim is to put the system back to a lower degradation level, in order to enlarge its period of good functioning, before it is replaced. The condition of the system after a maintenance action depends on the maintenance efficiency, which typically lies between the two extreme cases: As Bad As Old (ABAO) and As Good As New (AGAN) repairs. Mimicking an ARA1 model for recurrent events [1], the maintenance effect is here modeled through a first-order Arithmetic Reduction of Deterioration (ARD1) or of Age (ARA1), which allows to range from the ABAO to the AGAN case. An ARD1 maintenance action reduces the system degradation accumulated from the last maintenance action whereas an ARA1 one reduces its age. Note that in the case of a convex shape function for the Gamma process, an ARA1 action reduces the rate of deterioration together with the system age whereas this rate is not affected by an ARD1 action. In that case, both models hence deeply differ and they will not model imperfect repairs with similar maintenance actions. The aim of the present study is to compare the two models and try to understand the main differences between them.
Setting $(Y_t)_{t\geq 0}$ and $(Z_t)_{t\geq 0}$ to be the processes describing the deterioration levels of the system submitted to successive ARD1 and ARA1 actions (respectively) at periodic times, we prove (among other results) that, when the shape function of the gamma process is concave (resp. convex), then Y_t is smaller (resp. larger) than Z_t with respect to the increasing convex (resp. concave) order for all $t \geq 0$. In the case of a homogeneous gamma process, then Y_t is always larger than Z_t with respect to the convex order.

Keywords: Reliability, Gamma process, Imperfect repair, Convex (concave) order

AMS Classification: 90B25, 60E15, 60K10

References

[1] L. Doyen and O. Gaudoin, *Classes of imperfect repair models based on reduction of failure intensity or virtual age*, Reliab. Eng. Syst. Saf. **84** (2004), no. 1, 45–56.

[2] J.M. Van Noortwijk, *A survey of the application of Gamma processes in maintenance*, Reliab. Eng. Syst. Saf. **94** (2009), no. 1, 2–21.

[1]Laboratoire de Mathématiques et de leurs Applications - Pau (UMR CNRS 5142)
Université de Pau et des Pays de l'Adour (France)
email: sophie.mercier@univ-pau.fr

[2]Department of Mathematics, Escuela Politécnica
University of Extremadura (Spain)
email: inmatorres@unex.es