THE REGULAR POLYGON PROBLEM OF $N + 1$ BODIES: the past, the present and the future

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SUMMARY

The regular polygon problem of $N + 1$ bodies (also known as the ring problem of $N + 1$ bodies), is a theoretical model in the area of Celestial Mechanics, aiming at approaching the dynamical behavior of a small body, either natural or artificial, under the effect of N big bodies (primaries) the $\nu = N - 1$ of which have equal masses and are located at the vertices of a regular polygon, while the Nth body with a different mass is located at the center of mass of the system. The particular model has mainly been studied during the last 17 years both in its original classical form and in various versions, by an increasing number of researchers. The configuration of the N big bodies was based on a model proposed by Maxwell in the middle of the 19th century who tried to explain the already known rings of Saturn. Later that model was used to simulate planetary systems with co-orbital satellites (moons). At this point let us note that quasi co-orbital systems have also been observed in the outer planets of our solar system.

As is the case with every new problem, its investigation has passed through various stages or steps. First there is the genesis (foundation) of the problem (model). The original inspiration and the prime ideas led to the creation of a realizable and consistent model described by a set of proper equations, with the proper parameters taken into account and under the suitable assumptions or restrictions. Next, is the coming of age of the problem; the systematic investigation of the main dynamical properties, the classification of the existing solutions and their parametric variation, as well as the discovery of new properties. Then, the time of maturity comes. New ideas based on accumulated experience led to new improved and efficient methods of investigation, which unveiled and uncovered little by little several parts of the hidden secret world of the system.

My talk remains within this frame and can be considered as a mixture of the principal historical milestones of the problem, together with a brief reference to the main results obtained so far, an overview of the various versions of the system presented in the past few years and an exposition of the future perspective and proposals of new problems based on this model. In every topic, there has been an effort to mention as many scientists as possible, who contributed in various parts of the problem, each in their own way, adding new knowledge, fresh ideas and valuable experience to its investigation.

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