

VMS error estimation for computational fluid mechanics

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SUMMARY

The variational multiscale theory has been used to successfully derive a posteriori error estimators for solutions computed with finite element methods. By decomposing the exact solution into coarse scale (finite element solution) and fine scale (error), according to the nature of the residuals, the solution error can be split into two components: element interior residuals and inter-element jumps. The relationship between these residuals (coarse scales) and the error components (fine scales) is established, yielding to a very simple model. In particular, the pointwise error is modeled as a linear combination of bubble functions for the element interior residuals and free-space Green's functions for the inter-element jumps. The numerical error is studied for the standard Galerkin and SUPG methods with application to the heat equation, the Helmholtz equation and the convection-diffusion equation.

Keywords: finite elements, variational multiscale theory, a posteriori error estimation

AMS Classification: 65L11,65M60

References

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