

On the convergence of a finite difference scheme for a time fractional-diffusion equation

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SUMMARY

The fractional-diffusion equation with a Caputo time derivative of order $\delta \in (0, 1)$ is considered. The solutions of such problems are shown to be nonsmooth in general near the initial time $t = 0$. A new analysis of a standard finite difference method on a uniform and graded mesh for this problem is given, taking into account the weak singularity of the solution at $t = 0$. This analysis includes new stability and consistency bounds. The final convergence result shows how the grading of the mesh affects the order of convergence of the difference scheme and reveals the optimal mesh grading. Numerical results are presented that confirm the sharpness of the analysis.

Keywords: Caputo derivative, time-fractional, weak singularity, finite difference method, graded mesh

AMS Classification: 65M06, 35B65

References

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