Fourteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 12–15th 2016

Stokes equations with Navier boundary condition

Ghosh, Amrita and **Amrouche, Chérif**

SUMMARY

We are interested in the study of the theory of incompressible fluids in the bounded domain of $\mathbb{R}^3$. We consider here the stationary Stokes equations :

$$-\Delta \boldsymbol{u} + \nabla \pi = \boldsymbol{f} \quad \text{and} \quad \operatorname{div} \boldsymbol{u} = 0 \quad \text{in } \Omega, \tag{1}$$

where Ω is an open, bounded and connected set in $\mathbb{R}^3$ of class $C^{2,1}$. To study this problem, it is necessary to add appropriate boundary conditions. Note that in most of the mathematical works on these type of equations, classical Dirichlet boundary conditions for the vector field has been considered. However, for certain cases, this condition is not always realistic and hence it is necessary to introduce other type of boundary conditions. In this context, Navier [1] proposed in 1827, a boundary condition in which there is a layer of fluid close to the boundary allowing the fluid to slip and the tangential component of the strain tensor should be proportional to the tangential component of the fluid velocity on the boundary :

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0 \quad \text{and} \quad 2\left[\mathbf{D}(\boldsymbol{u})\boldsymbol{n}\right]_{\tau} + \alpha \boldsymbol{u}_{\tau} = \mathbf{0} \quad \text{on } \partial\boldsymbol{\Omega}. \tag{2}$$

Here α is the coefficient of friction which is a function depending on the geometry of the domain and $\mathbf{D}(\boldsymbol{u})$ denotes the deformation tensor associated to the velocity field u, defined as :

$$\mathbf{D}(\boldsymbol{u}) = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\top}).$$

The objective of this presentation is to solve the Stokes system 1 with the boundary conditions 2. We will deal with different geometric conditions of the domain and demonstrate the existence, uniqueness and regularity of the solution. Firstly, we consider the Hilbert case. And next, we generalize the study for L^p, with $1 < p < \infty$.

Keywords: Stokes equations, Navier boundary conditions, fluid mechanics

AMS Classification: -

References

[1] C.L.M.H. NAVIER. Sur les lois d'équilibre et du mouvement des corps élastiques, Mem. Acad. R. Sci. Inst. 6, France, vol. 7, 375–393 (1827).

[2] Chérif Amrouche, Rejaiba Ahmed. L^p theory for Stokes and Navier-Stokes equations with Navier boundary condition, J. Differential equations, 256(4):1515-1547, 2014.

[3] V.A. Solonnikov, V.E. Ščadilov, A certain boundary value problem for the stationary system of Navier-Stokes equations, Tr. Mat. Inst. Steklova 125 (1973) 196-210.

UMR CNRS 5142 Bâtiment IPRA, Avenue de l'Université - BP 1155
64013 PAU CEDEX
email: amrita.ghosh@univ-pau.fr
email: cherif.amrouche@univ-pau.fr