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Particle methods for non-linear Stokes equations coupled to the transport of heterogeneity

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SUMMARY

This talk presents numerical methods recently developed for complex creeping flows. A Stokes fluid is considered in interaction with moving obstacles. This fluid is non-Newtonian and its rheology depends on the fluid's composition, this heterogeneity is transported by the flow. To solve the resulting 3D set of non-linear PDEs [6] dedicated numerical algorithms have been developed they are based on: (i) a splitting algorithm [5] and an hybrid grid-particles discretization of the unknown to use a suitable technique for each phenomena (Lagrangian for the transport and Eulerian for the diffusion) ; (ii) the penalization method [1] to handle the fluid-structure interaction coupled to an algebraic perturbation technique based on the Sherman-Morrison-Woodbury formula [3] ; (iii) an iterative projection to compute accurately both incompressibility and spurious velocity [2] ; (iv) fast FFT-based solvers to guarantee a quasi-linear computational cost [7].

These numerical methods have been used to compute the mucociliary clearance (the natural mucus flow propelled by micro-metric cilia) and porous media flows. Some simulation examples will be presented in this presentation. The reduced computational cost of the algorithms enables to study the influence of the modeling parameters on the flow [4].

Keywords: Particle methods, Three-dimensional heterogeneous flows, Stokes equations, Complex geometry, Mucociliary clearance, Porous media.

AMS Classification: 65M06, 65M12, 65M25, 68U20, 76D07, 76D17, 76Z05, 92B05

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