Quotient surface singularities and lattice point counting
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SUMMARY

Given a normal surface $X$ the generalized Riemann-Roch formula ([7, 9, 3, 2, 1])

$$\chi(O_X(D)) = \chi(X) + \frac{1}{2} D \cdot (D - K_X) + R_X(D),$$

allows one to relate the Euler characteristics of $O_X(D)$ and $O_X$ via the canonical divisor $K_X$ and a correcting term $R_X(D)$. Such a term is the main object of this talk.

The invariant $R_X : \text{Cl}(X) \to \mathbb{Q}$ only depends on the rational divisor class in $X$, that is, the quotient of the Weil group by Cartier divisors and can be defined as a sum of associated invariants at the singular points of $X$ ([4]).

For this reason we only consider the local case. Assume $X = \mathbb{C}^n/G$, the group $\text{Cl}(X)$ is naturally isomorphic to the group of characters of $G$, that is, $G^\vee = \text{Hom}(G, \mathbb{C}^*)$. Following M.Reid’s notation [10] if $D_1, D_2 \in O_X(a)$ then $R_X(D_1) = R_X(D_2) =: R_X(a)$.

We will show that not every rational divisor class contains curvettes, in particular their generic divisors might not be irreducible. We will describe generic divisors and will obtain formulas for $R_X$ using this description.

In the case of cyclic quotient singularities $X = \mathbb{C^n}/\mathbb{Z}_d(1, p)$, this description can be given via the Hirzebruch-Jung continued fraction decomposition $[q_1, \ldots, q_n]$ of $\frac{p}{d}$ and the use of the greedy algorithm of $a \in \mathbb{Z}_d = G^\vee$ with respect to $[q_1, \ldots, q_n]$.

Possible applications of these results are lattice point counting formulas for rational polytopes [5] and Kouchnirenko’s ([8, 6]) formulas for the Milnor number of a curve on a normal surface singularity.

References


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