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Quotient surface singularities and lattice point counting

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SUMMARY

Given a normal surface X the generalized Riemann-Roch formula ([7, 9, 3, 2, 1])

$$\chi(\mathcal{O}_X(D)) = \chi(X) + \frac{1}{2} D \cdot (D - K_X) + R_X(D),$$

allows one to relate the Euler characteristics of $\mathcal{O}_X(D)$ and $\mathcal{O}_X$ via the canonical divisor K_X and a correcting term $R_X(D)$. Such a term is the main object of this talk.

The invariant $R_X : \mathrm{Cl(X)} \to \mathbb{Q}$ only depends on the rational divisor class in X, that is, the quotient of the Weil group by Cartier divisors and can be defined as a sum of associated invariants at the singular points of X ([4]).

For this reason we only consider the local case. Assume $X = \mathbb{C}^n/G$, the group Cl(X) is naturally isomorphic to the group of characters of G, that is, $G^\vee = Hom(G, \mathbb{C}^*)$. Following M.Reid's notation [10] if $D_1, D_2 \in \mathcal{O}_X(a)$ then $R_X(D_1) = R_X(D_2) =: R_X(a)$.

We will show that not every rational divisor class contains curvettes, in particular their generic divisors might not be irreducible. We will describe generic divisors and will obtain formulas for R_X using this description.

In the case of cyclic quotient singularities $X = \frac{1}{d}(1, p)$, this description can be given via the Hirzebruch-Jung continued fraction descomposition $[q_1, ..., q_n]$ of $\frac{p}{d}$ and the use of the *greedy* algorithm of $a \in \mathbb{Z}_d = G^\vee$ with respect to $[q_1, ..., q_n]$.

Possible applications of these results are lattice point counting formulas for rational polytopes [5] and Kouchnirenko's ([8, 6]) formulas for the Milnor number of a curve on a normal surface singularity.

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