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Convergence analysis of the upwind scheme for the transport equation with boundary data

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SUMMARY

In this talk, I consider the upwind finite volume scheme on general unstructured grids, for the transport equation taking into account inflow boundary data:

$$\begin{cases} \partial_t \rho + v(t,x) \cdot \nabla_x \rho = 0, & t \in (0,T), \quad x \in \Omega, \\ \rho(0,x) = \rho_0(x), & x \in \Omega, \\ \rho(t,x) = \rho^i(t,x), & t \in (0,T), \quad x \in \Gamma, \text{ where } v(t,x) \cdot n(x) < 0. \end{cases} \tag{P}$$

I will mainly present two results:

- A uniform-in-time strong convergence in any L^p, $p < +\infty$, of the approximate solution under weak regularity assumptions on the advection field v and on the data, see [1]. Actually the assumptions we consider are the same as in the DiPerna-Lions theory, for which I will recall the available trace theory, see for instance [2].
- An error estimate of order $1/2$ in the L^1 norm, under slightly stronger regularity assumptions on the data that lead to BV weak solutions. This result is a joint work with Nina Aguillon (Lab. Jacques-Louis Lions, Université Paris 6), see [3].

Keywords: transport equation, renormalized solutions, upwind scheme

References

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