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Minimal k-partition for the p-norm of the eigenvalues

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SUMMARY

In this talk, we are interested in the analysis of minimal partitions and their relations with nodal domains of eigenfunctions for suitable operators. For any simply connected domain $\Omega \subset \mathbb{R}^2$, the optimization problem considered here consists in minimizing among the k-partitions of Ω the energy

$$\Lambda_k(\mathcal{D}) = \max\{\lambda(D_j), 1 \leq j \leq k\}, \tag{1}$$

where $\mathcal{D} = (D_1, \dots, D_k)$ is a k-partition of Ω and $\lambda(D_j)$ denotes the first eigenvalue of the Dirichlet-Laplacian on D_j.
For this problem, we present some theoretical and numerical results and exhibit candidates to be minimal (cf. [2] for a survey about this topics).
A generalization to p-minimal k-partiton where the ℓ^∞-norm defining $\Lambda(\mathcal{D})$ is replaced by the ℓ^p-norm is also considered (cf. [1]) from a numerical point of view, using [3]. The p-energy of the k-partition $\mathcal{D} = (D_1, \dots, D_k)$ is thus given by

$$\Lambda_{k,p}(\mathcal{D}) = \left(\frac{1}{k} \sum_{j=1}^{k} \lambda(D_j)^p \right)^{1/p}.$$

Keywords: Optimal partitions, shape optimization, Dirichlet-Laplacian, projected gradient algorithm

AMS Classification: 49Q10, 35J05, 65K10, 65N06, 65N25

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