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Convergence of monotone finite volume schemes for hyperbolic scalar conservation laws with a multiplicative stochastic force

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SUMMARY

We are interested in the Cauchy problem for a nonlinear hyperbolic scalar conservation law in d space dimensions with a multiplicative stochastic perturbation of type:

$$\left\{\begin{array}{rcll} du + \operatorname{div}\left[\vec{v}(x,t)f(u)\right]dt &=& g(u)dW & \text{in } \Omega \times \mathbb{R}^d \times (0,T), \\ u(\omega,x,0) &=& u_0(x), & \omega \in \Omega,\ x \in \mathbb{R}^d, \end{array}\right. \tag{1}$$

where div is the divergence operator with respect to the space variable, d is a positive integer, $T > 0$ and $W = \{W_t, \mathcal{F}_t; 0 \leqslant t \leqslant T\}$ is a standard adapted one-dimensional continuous Brownian motion defined on the classical Wiener space $(\Omega, \mathcal{F}, P)$. I will present in this talk the discretization of Problem (1) by monotone finite volume schemes.

Firstly, I will introduce the well-posedness theory for solutions of such kind of stochastic problems. Then, the main part of the talk will be devoted to the study of the monotone numerical scheme used to approximate the solution of (1). I will show that under a stability condition on the time step, one is able to show the convergence of the finite volume approximation towards the unique stochastic entropy solution of the equation.

Keywords: Stochastic PDE, first-order hyperbolic equation, Itô integral, multiplicative noise, finite volume method, monotone scheme, Godunov scheme, entropy solution, Kruzhkov smooth entropy, Young measures, ...

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