Thirteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 15–18th 2014

A stochastic $\Delta_{p(\cdot)}$ problem

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Minisymposium : nonlinear PDE

SUMMARY

In this communication, we are interested in the stochastic partial differential problem:

$$(P): \quad \begin{array}{cc} du - \triangle_{p(\cdot)}u \; dt = h(\cdot, u)dw & \text{in } \Omega \times (0,T) \times D \\ u = 0 & \text{on } \Omega \times (0,T) \times \partial D \\ u(0,\cdot) = u_0 & \text{in } \Omega \times D \end{array}$$

where we assume that:

· T is a positive number, $D \subset \mathbb{R}^d$ is a bounded domain with a Lipschitz boundary and $w = \{w_t, \mathcal{F}_t; 0 \leq t \leq T\}$ denotes a standard adapted one-dimensional continuous Brownian motion, defined on the classical Wiener space $(\Omega, \mathcal{F}, P)$;

· h is a Carathéodory function in the sense that:
for any $\lambda \in \mathbb{R}$, $h(\cdot, \lambda) \in N^2_W(0, T, L^2(D))$, the space of predictable processes with values in $L^2(D)$, and, $P \otimes \mathcal{L}^{d+1}$-a.e., $\lambda \in \mathbb{R} \to h(t, x, \omega, \lambda) \in \mathbb{R}$ is continuous. Moreover, h is a Lipschitz-continuous function of the variable λ, uniformly with respect to the other variables;

· the variable exponent is a measurable function $p : Q_T \to (1, \infty)$ satisfying
$1 < p^- = \operatorname{ess\,inf}_{(s,y)\in Q_T} p(s, y) \leq p(t, x) \leq p^+ = \operatorname{ess\,sup}_{(s,y)\in Q_T} p(s, y) < \infty$, and $\Delta_{p(\cdot)}u$ denotes the formal differential operator div $\left[|\nabla u|^{p(t,x)-2}\nabla u\right]$;

· the initial condition $u_0 \in L^2(D)$ and homogeneous Dirichlet boundary conditions are required.

Keywords: Stochastic forcing, p-Laplace, variable exponents

AMS Classification: 35K92, 46E30, 60H15

References

[1] C. Bauzet, G. Vallet, P. Wittbold, A. Zimmermann, On a $p(t, x)$-Laplace evolution equation with a stochastic force. Stoch PDE: Anal Comp, DOI 10.1007/s40072-013-0017-z

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