

Design of a multigrid solver for mimetic finite differences by using a finite element framework

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SUMMARY

Our focus in this work is the efficient multigrid solution of the algebraic systems of equations resulting from the mimetic finite difference (MFD) schemes for elliptic partial differential equations. Some of the advantages of these discretizations are the following: they work on general unstructured and irregular grids not necessarily aligned with coordinate axes, and such schemes result in discrete grid operators which satisfy the compatibility conditions (exact sequence properties) connecting grad, div and curl operators on the continuous level. Here, we show how such MFD schemes can be derived using standard finite element spaces in $H(\text{curl})$. In this way, using the finite element framework, we are able to analyze the convergence of the MFD discretizations and design multigrid methods for the solution of the resulting linear systems. We propose, and, via the local Fourier analysis (LFA) framework we also analyze geometric multigrid algorithms for such problems. Finally, we present several numerical tests which demonstrate the efficiency of the proposed multigrid methods and the sharpness of the LFA estimates of the convergence rate.

Keywords: mimetic finite differences, multigrid methods, local Fourier analysis, finite element methods,...

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