Thirteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 15–18th 2014

On the regularity of the Q-Tensor model depending on the data[†]

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SUMMARY

The coupled Navier-Stokes and Q-Tensor system is one of the models used to describe the behavior of the nematic liquid crystals, an intermediate phase between crystalline solids and isotropic fluids. These equations model the dynamics of the fluid via velocity and pressure $(\boldsymbol{u}, p)$ and the orientation of the molecules via a tensor Q. A review on the existence of weak solutions, maximum principle and a uniqueness criteria can be seen in [1] (the corresponding Cauchy problem in the whole $\mathbb{R}^3$ is analysed in [4]). However, the regularity of such solutions is only analysed under some restrictive conditions: large viscosity or periodic boundary conditions.

In this talk, we study two types of "more regular solutions" for the Q-Tensor model, which are not equivalent: one inherited from the usual strong solution for the Navier-Stokes equations, and another one where $(\boldsymbol{u}, Q)$ and $(\partial_t \boldsymbol{u}, \partial_t Q)$ have weak regularity. The reason to introduce new concepts of "more regular solutions" is the impossibility of dealing with the boundary terms in order to obtain estimates in strong norms when boundary conditions different from periodic ones are imposed. For both types of regular solution, their existence (and uniqueness) is local in time.

Some regularity criteria for $(\boldsymbol{u}, Q)$ will also be given. In the particular case of Neumann boundary conditions for Q, the regularity criteria only must be imposed for the velocity $\boldsymbol{u}$.

Keywords: Q-Tensor, weak regularity, strong regularity, regularity criteria

AMS Classification: 35B65, 35K51, 35Q35, 76A15, 76D03

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[2] F. Guillén-González and M. A. Rodríguez-Bellido *Weak time regularity and uniqueness for a Q-Tensor model.* Submitted.

[3] F. Guillén-González and M. A. Rodríguez-Bellido *A uniqueness and regularity criterium for Q-tensor models with Neumann boundary conditions.* Submitted.

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[†]The authors are partially financed by the MINECO project MTM2012-32325 (Government of Spain).