On the quantile functions of the Log–Lindley distribution and a discrete Lindley distribution†

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SUMMARY

The Lindley distribution was introduced by the British statistician Dennis V. Lindley (1923–2013) in the context of Bayesian Statistics. Jodrá [3] shown that this probability distribution has the variate generation property (vgp), that is, the quantile function (qf) can be given in closed form. The qf of a random variable $T$ is defined by $Q_T(u) := \inf\{t \in \mathbb{R} : F_T(t) \geq u\}$, $0 < u < 1$, where $F_T(t) := P(T \leq t)$. The vgp is a remarkable property since it implies that random samples can be computer-generated by means of the inverse transform method.

Using the Lindley distribution, Gómez-Déniz et al. [2] have derived the Log–Lindley distribution, which has been proposed as an alternative to the beta distribution, and Gómez-Déniz and Calderín-Ojeda [1] have introduced a discrete Lindley distribution by discretizing the Lindley distribution, which has been proposed as an alternative to the Poisson model.

In this work, we show that the Log–Lindley distribution ($X$) and the aforementioned discrete Lindley distribution ($Y$) have the vgp. More specifically, their qfs can be expressed in closed form in terms of the Lambert $W$ function as follows

$$Q_X(u; \lambda, \sigma) = \exp\left\{1 + \frac{\lambda \sigma}{\sigma}\right\} \exp\left\{\frac{1}{\sigma} W_{-1}\left(-\frac{u(1 + \lambda \sigma)}{\exp\left\{1 + \lambda \sigma\right\}}\right)\right\}, \quad 0 < u < 1, \quad \lambda \geq 0, \sigma > 0,$$

$$Q_Y(u; \lambda) = \left\lceil -2 + \frac{1}{\log \lambda} + \frac{1}{\log \lambda} W_{-1}\left((u - 1)(1 - \log \lambda)\lambda e^{-1}\right)\right\rceil, \quad 0 < u < 1, \quad 0 < \lambda < 1,$$

where $W_{-1}$ denotes the negative branch of the Lambert $W$ function and $\lceil \cdot \rceil$ stands for the ceiling of a real number. The vgp reinforces the interest in these models as alternatives to the beta and Poisson distributions since these classical distributions do not have that property.

Keywords: Lindley distribution, Lambert $W$ function, simulation

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References


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