Thirteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 15–18th 2014

On a logarithmic integral and order statistics of the Weibull-geometric distribution[†]

P. Jodrá[1], **M.D. Jiménez-Gamero**[2]

SUMMARY

For any $n = 1, 2, \ldots$ and any real numbers $a > -1$, $s \geq 1$ and $z > -1$, we show that

$$\int_0^1 \frac{u^a \log^{s-1}(1/u)}{(1+zu)^{n+1}}\, du = \frac{\Gamma(s)}{\Gamma(n+1)} \sum_{j=0}^{n} R_n^j(a-n+1,1)\Phi(-z, s-j, a+1),$$

where Φ denotes the Lerch transcendent function and R_n^j denotes the generalized Stirling numbers of the first kind (cf. Mitrinović [4]). In particular, for $a = 0, 1, \ldots, n-1$, $n = 1, 2, \ldots$, $s \geq 1$ and $z \in (-1, 0) \cup (0, \infty)$, we also prove the following

$$\int_0^1 \frac{u^a \log^{s-1}(1/u)}{(1+zu)^{n+1}}\, du = \frac{\Gamma(s)}{(-z)^{a+1}\Gamma(n+1)} \sum_{j=1}^{n} R_n^j(a-n+1,1)\mathrm{Li}_{s-j}(-z),$$

where Li stands for the polylogarithm function. These results extend those of Medina and Moll [3].

As a consequence of the above results, we provide explicit expressions for the non-central moments of the order statistics from the Weibull-geometric distribution (cf. Barreto-Souza et al. [1] and also Jodrá and Jiménez-Gamero [2]), which is a generalization of the exponential-geometric distribution.

Keywords: Logarithmic integral, Polylogarithms, Weibull-geometric distribution

AMS Classification: 33E20, 11M35, 60E99

References

[1] W. Barreto-Souza, A. Lemos de Morais and G.M. Cordeiro. The Weibull-geometric distribution. *J. Stat. Comput. Simul.* **81**(5), 645–657, 2011.

[2] P. Jodrá and M.D. Jiménez-Gamero. On a logarithmic integral and the moments of order statistics from the Weibull-geometric and half-logistic families of distributions. *J. Math. Anal. Appl.* **410**(2), 882–890, 2014.

[3] L.A. Medina and V.H. Moll. A class of logarithmic integrals. *Ramanujan J.* **20**(1), 91–126, 2009.

[4] D.S. Mitrinović. Sur une classe de nombres reliés aux nombres de Stirling. *C. R. Acad. Sci. Paris* **252**, 2354–2356, 1961.

[1]Dpto. de Métodos Estadísticos, Universidad de Zaragoza
C/ María de Luna 3, 50018 Zaragoza, Spain
E-mail: pjodra@unizar.es

[2]Dpto. de Estadística e Investigación Operativa, Universidad de Sevilla
Avd. Reina Mercedes s.n, 41012 Sevilla, Spain
E-mail: dolores@us.es

[†]Acknowledgements. This work has been funded by DGA (Grupo consolidado PDIE).