

## A parabolic $\Delta_{p(\cdot)}$ problem

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### SUMMARY

In this communication, we are interested in the parabolic partial differential equation involving the  $p(x)$ -laplacian:

$$(P_T) : \begin{cases} \frac{du}{dt} - \Delta_{p(x)}u = f(x,u) & \text{in } Q_T := (0, T) \times \Omega \\ u = 0 & \text{on } (0, T) \times \partial\Omega \\ u(0, \cdot) = u_0 & \text{in } \Omega \end{cases}$$

where we assume that  $T > 0$  and  $\Omega$  is a smooth domain of  $\mathbb{R}^d$  ( $d \geq 2$ ) and  $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$  is a Carathéodory function locally Lipschitz with respect to the second variable uniformly in the first. The variable exponent is a measurable function  $p : \Omega \rightarrow (1, \infty)$  satisfying

$$1 < p^- = \operatorname{ess\,inf}_{\Omega} p(\cdot) \leq p(\cdot) \leq p^+ = \operatorname{ess\,sup}_{\Omega} p(\cdot) < d$$

and  $\Delta_{p(\cdot)}u$  denotes the formal differential operator  $\operatorname{div} [|\nabla u|^{p(x)-2}\nabla u]$ .

We study the problem  $(P_T)$  under growth assumptions on the nonlinearity  $f$  and on the initial data  $u_0$ , under this conditions we obtain existence and stabilization results.

**Keywords:** parabolic equation,  $p(\cdot)$ -Laplace, variable exponents

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