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Journal of Approximation Theory 128 (2004) 100–101

JOURNAL OF Approximation Theory

http://www.elsevier.com/locate/jat

Short note

A note on the error bound for the remainder of an asymptotic expansion of the double gamma function

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Received 27 October 2003; accepted in revised form 18 February 2004

Using a recent result [2], we can improve the error bound for an asymptotic expansion for the double gamma function $G(z)$ given in [1, Theorem 2] when $|\text{Arg}(z)|<\pi/2$. This new bound is simpler as well as more accurate.

Theorem 1. *For* $|Arg(z)|<\pi/2$, *an error bound for the remainder* $R_N(z)$ *in the expansion* [1, *Theorem* 1] *of* $\log G(z+1)$ *is given by* [1, (9)]

$$|R_N(z)| \leqslant \frac{|B_{2N+2}|}{2N(2N+1)(2N+2)(\text{Re}(z))^{2N}}, \quad N = 1, 2, 3, \ldots \tag{1}$$

Proof. For $|\text{Arg}(z)|<\pi/2$ we can take $\varphi = 0$ in formula [1, (11)]. In [2] the author proves that the inequality $(-1)^N r_{2N+2}(x) \geqslant 0$, for $r_{2N+2}(x)$ given in [1, (9)], holds when $x \geqslant 0$. Then, $r_{2N+2}(x)$ verifies the error test for all positive x and the bound given in [1, (13)] is valid for $0 \leqslant x < \infty$. Using this bound in [1, (11)] we obtain the result.

In Table 1, we compare the relative error bounds given in [1, Theorem 2] (namely "old") with (1) (namely "new"). This new bound is in particular more accurate for small values of z. The new bound (1) has analytical as well as computational advantages for lower values of z.

This note has been stimulated by conversations with López and Pedersen at the seventh international symposium on Orthogonal Polynomials, Special Functions and Applications in Copenhagen, August 2003.

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0021-9045/$ - see front matter
doi:10.1016/j.jat.2004.02.002

Table 1
Approximation supplied by [1, Theorem 1] taking $z \in$ and error bounds given by [1, Theorem 2] and (new bound)

z	$I_2(z)$	Relat. err. (1st ord)	Old relat. er. bound	New relat. er. bound	Relat. err. (2nd ord)	Old relat. er. bound	New relat. er. bound
1	−0.0012455	0.115	2.24	0.16	0.0442	13.4	0.08
2	−0.0003361	0.0332	0.0425	0.037	0.00374	0.032	0.0046
5	−0.0000552	0.00564	0.00574	0.00574	0.000110	0.000115	0.0001149
10	−0.0000139	0.00142	0.00143	0.00143	7.009e-6	7.18e-6	7.18e-6
20	−3.471e-6	0.0003568	0.0003572	0.0003572	4.45e-7	4.47e-7	4.47e-7
50	−5.55552e-7	5.713e-5	5.715e-5	5.715e-5	1.142e-8	1.143e-8	1.143e-8
100	−1.38887e-7	1.42852e-5	1.42859e-5	1.42859e-5	7.142e-10	7.143e-10	7.143e-10

References

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[2] H.L. Pedersen, On the remainder in an asymptotic expansion of the double gamma function, manuscript.