1 Continuous Dual Hahn to Meixner

1. Asymptotic expansion for large a, b y c:

$$\frac{S_n(x^2;a,b,c)}{(a+b)_n n!} = \sum_{k=0}^{n} c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C),$$

where $C \neq 0, 1$ is an arbitrary constant,

$$X = \frac{C^2}{1-C}\left[p_1(x)^2 + p_1(x) - 2p_2(x)\right], \quad A = (1+C)p_1(x) + Cp_1(x)^2 - 2Cp_2(x), \tag{1}$$

with $p_1(x)$ and $p_2(x)$

$$p_1(x) = c + \frac{ab - x^2}{a+b},$$

$$p_2(x) = \frac{c(c+1)}{2} + \frac{abc}{a+b} + \frac{ab(1+a)(1+b) - [c + 2ab + (1+c)(1+2(a+b))]\,x^2 + x^4}{2(a+b)(1+a+b)}$$

and

$$f(x,w) = \left(1 - \frac{w}{C}\right)^{-X}(1-w)^{X+A}(1-w)^{-c+ix}\,{}_2F_1\left(\left.\begin{matrix} a+ix, b+ix \\ a+b \end{matrix}\right| w\right).$$

2. Asymptotic property:

$$c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C) = \mathcal{O}\left(a^{n+\left[\frac{k}{3}\right]-k}\right)$$

when a, $b \to \infty$ uniformly in c with $a \sim b$ and c/a bounded.

3. Limit:

$$\lim_{a\to\infty}\left[\frac{S_n(\tilde{x};a,a,\tilde{c})}{\left(c + \frac{C-1}{C}\frac{x}{1+2a}\right)_n (2a)_n}\right] = M_n(x;c,C),$$

where

$$\tilde{x} = -a^2 - 2\frac{a}{C}\sqrt{2ax(1-C)}, \qquad \tilde{c} = -a + c - \frac{1-C}{C}x - \frac{2}{C}\sqrt{2ax(1-C)}.$$

2 Continuous Hahn to Meixner

1. Asymptotic expansions for large a and b:

$$\frac{(2a+2b-1)_n p_n(x;\alpha,\beta,\overline{\alpha},\overline{\beta})}{(2a)_n(a+b+i(c-d))_n i^n} = \sum_{k=0}^{n} c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C),$$

where $C \neq 0, 1$ is an arbitrary constant, X and A are given in (1), $p_1(x)$ and $p_2(x)$,

$$p_1(x) = \frac{i\,[1 - 2(a+b)]\,[b(c+x) + a(d+x)]}{a[(a+b) + i(c-d)]},$$

$$\begin{aligned} p_2(x) &= \frac{p_1(x)}{i(1+2a)[1+a+b+i(c-d)][b(c+x)+a(d+x)]} \\ &\times\Big\{(a+b)\Big[(1+2b)(c+x)(d+x+b(c+x)) + a^2(b - 2(d+x)^2) \\ &+ a[b^2 - (d+x)(2c+d+3x) - b(-1+4dx+4x^2+4c(d+x))]\Big]\Big\} \end{aligned}$$

and

$$f(x,w) = \left(1 - \frac{w}{C}\right)^{-X}(1-w)^{X+A}(1-w)^{1-2(a+b)}\,{}_3F_2\left(\left.\begin{matrix} a+b-\frac{1}{2}, a+b, a+i(c+x) \\ 2a, a+c+i(b-d) \end{matrix}\right| -\frac{4w}{(1-w)^2}\right).$$

2. Asymptotic property:

$$c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C) = \mathcal{O}\left(a^{n+\left[\frac{k}{3}\right]-k}\right) \quad \text{when} \quad a,\, b \to \infty \quad \text{with} \quad a \sim b.$$

3 Hahn to Meixner

1. Asymptotic expansions for large a, b and N:

$$\frac{(a+b+1)_n}{n!} Q_n(x;a,b,N) = \sum_{k=0}^{n} c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C),$$

where $C \neq 0, 1$ is an arbitrary constant, X and A are given in (1), $p_1(x)$ and $p_2(x)$

$$p_1(x) = \frac{(1+a+b)\left[(1+a)N - (2+a+b)x\right]}{(1+a)N},$$

$$\begin{aligned} p_2(x) = \; & \frac{(2+3(a+b)+(a+b)^2)}{2N(N-1)(a+2)(a+1)} \times \Big\{ N^2(2+3a+a^2) \\ & -N(2+a)[1+a+(6+2a+2b)x] + (3+a+b)x[a-b+x(4+a+b)] \Big\} \end{aligned}$$

and

$$f(x,w) = \left(1 - \frac{w}{C}\right)^{-X} (1-w)^{X+A} (1-w)^{-1-a-b} {}_3F_2\left(\left. \begin{matrix} (a+b+1)/2, (a+b+2)/2, -x \\ a+1, -N \end{matrix} \right| -\frac{4w}{(1-w)^2} \right).$$

2. Asymptotic property:

$$c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C) = \mathcal{O}\left(a^{n-k}\right) \quad \text{when} \quad a,\, b,\, N \to \infty \quad \text{with} \quad a \sim b \sim N.$$

4 Dual Hahn to Meixner

1. Asymptotic expansion for large a and N:

$$\frac{(-N)_n R_n(\lambda(x);a,b,N)}{n!} = \sum_{k=0}^{n} c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C), \quad \lambda(x) \equiv x(x+a+b+1),$$

where $C \neq 0, 1$ is an arbitrary constant, X and A are given in (1), $p_1(x)$ and $p_2(x)$

$$p_1(x) = \frac{\lambda(x)}{a+1} - N,$$

$$p_2(x) = \frac{(N-x)(N-x-1)}{2} + \frac{x(b+x)\left[2(2+a)(x-N) + (b+x-1)(x-1)\right]}{2(a+1)(2+a)}$$

and

$$f(x,w) = \left(1 - \frac{w}{C}\right)^{-X} (1-w)^{X+A} (1-w)^{N-x} {}_2F_1\left(\left. \begin{matrix} -x, -b-x \\ a+1 \end{matrix} \right| w \right).$$

2. Asymptotic property:

$$c_k \frac{(A)_{n-k}}{(n-k)!} M_{n-k}(X;A,C) = \mathcal{O}\left(N^{n-k-1}\right) \quad \text{when} \quad a,\, N \to \infty \quad \text{with} \quad a \sim N.$$