

1 Continuous Dual Hahn to Krawtchouk

1. Asymptotic expansion for large a and b :

$$\frac{S_n(x^2; a, b, c)}{(a+b)_n n!} = \sum_{k=0}^n \binom{C}{n-k} c_k K_{n-k}(X; A, C),$$

where $A \neq 0, 1$ is an arbitrary constant,

$$X = \frac{A^2}{1-A} [p_1(x)^2 - p_1(x) - 2p_2(x)], \quad C = p_1(x) + \frac{X}{A}, \quad (1)$$

with $p_1(x)$ and $p_2(x)$

$$p_1(x) = c + \frac{ab - x^2}{a + b},$$

$$p_2(x) = \frac{c(c+1)}{2} + \frac{abc}{a+b} + \frac{ab(1+a)(1+b) - [c + 2ab + (1+c)(1+2(a+b))]x^2 + x^4}{2(a+b)(1+a+b)}$$

and

$$f(x, w) = \left(1 - \frac{1-A}{A}w\right)^{-X} (1+w)^{X-C} (1-w)^{-c+ix} {}_2F_1\left(\begin{matrix} a+ix, b+ix \\ a+b \end{matrix} \middle| w\right).$$

2. Asymptotic property:

$$\binom{C}{n-k} c_k K_{n-k}(X; A, C) = \mathcal{O}\left(a^{n+\lceil \frac{k}{3} \rceil - k}\right)$$

when $a, b \rightarrow \infty$ uniformly in c with $a \sim b$ and c/a bounded.

3. Limit:

$$\lim_{a \rightarrow \infty} \frac{S_n(\tilde{x}; a, a, \tilde{c})}{(\tilde{C})_n (2a)_n n!} = K_n(x; A, c),$$

where

$$\tilde{x} = -a^2 - 2\frac{a}{A}\sqrt{2a(2A^2c - 3Ax + x)}, \quad \tilde{C} = \frac{A^2(3+2a)c + x - A[(2a+1)c + 3x]}{A(A-1)(2a+1)},$$

$$\tilde{c} = -a + c - \frac{1}{A} \left[x + 2\sqrt{2a(2A^2c - 3Ax + x)} \right].$$

2 Continuous Hahn to Krawtchouk

1. Asymptotic expansions for large a and b :

$$\frac{(2a+2b-1)_n p_n(x; \alpha, \beta, \bar{\alpha}, \bar{\beta})}{(2a)_n (a+b+i(c-d))_n i^n} = \sum_{k=0}^n \binom{C}{n-k} c_k K_{n-k}(X; A, C),$$

where $A \neq 0, 1$ is an arbitrary constant, X and C are given in (1), $p_1(x)$ and $p_2(x)$

$$p_1(x) = \frac{i[1-2(a+b)][b(c+x)+a(d+x)]}{a[(a+b)+i(c-d)]},$$

$$\begin{aligned} p_2(x) &= \frac{p_1(x)}{i(1+2a)[1+a+b+i(c-d)][b(c+x)+a(d+x)]} \\ &\times \left\{ (a+b) \left[(1+2b)(c+x)(d+x+b(c+x)) + a^2(b-2(d+x)^2) \right. \right. \\ &\left. \left. + a[b^2 - (d+x)(2c+d+3x) - b(-1+4dx+4x^2+4c(d+x))] \right] \right\} \end{aligned}$$

and

$$f(x, w) = \left(1 - \frac{1-A}{A}w\right)^{-X} (1+w)^{X-C} (1-w)^{N-x} (1-w)^{1-2(a+b)} \\ \times {}_3F_2 \left(\begin{matrix} a+b-\frac{1}{2}, a+b, a+i(c+x) \\ 2a, a+c+i(b-d) \end{matrix} \middle| -\frac{4w}{(1-w)^2} \right).$$

2. Asymptotic property:

$$\binom{C}{n-k} c_k K_{n-k}(X; A, C) = \mathcal{O} \left(a^{n+\lceil \frac{k}{3} \rceil - k} \right) \quad \text{when } a, b \rightarrow \infty \quad \text{with } a \sim b.$$

3 Hahn to Krawtchouk

1. Asymptotic expansions for large a, b and N :

$$\frac{(a+b+1)_n}{n!} Q_n(x; a, b, N) = \sum_{k=0}^n \binom{C}{n-k} c_k K_{n-k}(X; A, C),$$

where $A \neq 0, 1$ is an arbitrary constant, X and C are given in (1), $p_1(x)$ and $p_2(x)$

$$p_1(x) = \frac{(1+a+b)[(1+a)N - (2+a+b)x]}{(1+a)N},$$

$$p_2(x) = \frac{(2+3(a+b) + (a+b)^2)}{2N(N-1)(a+2)(a+1)} \times \left\{ N^2(2+3a+a^2) \right. \\ \left. - N(2+a)[1+a+(6+2a+2b)x] + (3+a+b)x[a-b+x(4+a+b)] \right\}$$

and

$$f(x, w) = \left(1 - \frac{1-A}{A}w\right)^{-X} (1+w)^{X-C} (1-w)^{N-x} (1-w)^{-1-a-b} \\ \times {}_3F_2 \left(\begin{matrix} (a+b+1)/2, (a+b+2)/2, -x \\ a+1, -N \end{matrix} \middle| -\frac{4w}{(1-w)^2} \right).$$

2. Asymptotic property:

$$\binom{C}{n-k} c_k K_{n-k}(X; A, C) = \mathcal{O} \left(a^{n+\lceil \frac{k}{3} \rceil - k} \right) \quad \text{when } a, b, N \rightarrow \infty \quad \text{with } a \sim b \sim N.$$

4 Dual Hahn to Krawtchouk

1. Asymptotic expansion for large a and N :

$$\frac{(-N)_n R_n(\lambda(x); a, b, N)}{n!} = \sum_{k=0}^n \binom{C}{n-k} c_k K_{n-k}(X; A, C), \quad \lambda(x) \equiv x(x+a+b+1),$$

where $A \neq 0, 1$ is an arbitrary constant, X and C are given in (1), $p_1(x)$ and $p_2(x)$

$$p_1(x) = \frac{\lambda(x)}{a+1} - N,$$

$$p_2(x) = \frac{(N-x)(N-x-1)}{2} + \frac{x(b+x)[2(2+a)(x-N) + (b+x-1)(x-1)]}{2(a+1)(2+a)}$$

and

$$f(x, w) = \left(1 - \frac{1-A}{A}w\right)^{-X} (1+w)^{X-C} (1-w)^{N-x} {}_2F_1 \left(\begin{matrix} -x, -b-x \\ a+1 \end{matrix} \middle| w \right).$$

2. Asymptotic property:

$$\binom{C}{n-k} c_k K_{n-k}(X; A, C) = \mathcal{O} \left(N^{n+\lceil \frac{k}{3} \rceil - k} \right) \quad \text{when } a, N \rightarrow \infty \quad \text{with } a \sim N.$$