1 Continuous Dual Hahn to Jacobi

1. Asymptotic expansion for large a and b:

$$\frac{S_n(x^2;a,b,c)}{(a+b)_n n!}=\sum_{k=0}^{n} c_k P_{n-k}^{(A,C)}(X),$$

where $X\neq\pm1$ is an arbitrary constant,

$$\begin{aligned}
A &= \frac{1}{X+1}\left[2p_1(x)^2+p_1(x)+3p_1(x)X-4p_2(x)+X^2-X-2\right],\\
C &= \frac{1}{X-1}\left[2p_1(x)^2-p_1(x)+3p_1(x)X-4p_2(x)+X^2+X-2\right],
\end{aligned} \tag{1}$$

with $p_1(x)$ and $p_2(x)$ given in (**??**) and

$$f(x,w)=\frac{R(1+R-w)^A(1+R+w)^C}{2^{A+C}}(1-w)^{-c+ix}{}_2F_1\left(\left.\begin{matrix}a+ix,b+ix\\ a+b\end{matrix}\right|w\right),$$

with $R=\sqrt{1-2Xw+w^2}$.

2. Asymptotic property:

$$c_k P_{n-k}^{(A,C)}(X)=\mathcal{O}\left(a^{n+\left[\frac{k}{3}\right]-k}\right)\text{ when } a,\ b\to\infty \text{ uniformly in } c \text{ with } a\sim b \text{ and } c/a \text{ bounded.}$$

2 Continuous Hahn to Jacobi

1. Asymptotic expansion for large a and b:

$$\frac{(2a+2b-1)_n p_n(x;\alpha,\beta,\overline{\alpha},\overline{\beta})}{(2a)_n(a+b+i(c-d))_n i^n}=\sum_{k=0}^{n} c_k P_{n-k}^{(A,C)}(X),$$

where $X\neq\pm1$ is an arbitrary constant, A and C are given in (1) and

$$f(x,w)=\frac{R(1+R-w)^A(1+R+w)^C}{2^{A+C}(1-w)^{2(a+b)-1}}{}_3F_2\left(\left.\begin{matrix}a+b-\frac{1}{2},a+b,a+i(c+x)\\ 2a,a+c+i(b-d)\end{matrix}\right|-\frac{4w}{(1-w)^2}\right).$$

2. Asymptotic property:

$$c_k P_{n-k}^{(A,C)}(X)=\mathcal{O}\left(a^{n+\left[\frac{k}{3}\right]-k}\right)\quad\text{when}\quad a,\ b\to\infty\quad\text{with}\quad a\sim b.$$

3 Hahn to Jacobi

1. Asymptotic expansion for large a,b,N:

$$\frac{(a+b+1)_n}{n!}Q_n(x;a,b,N)=\sum_{k=0}^{n} c_k P_{n-k}^{(A,C)}(X),$$

where $X\neq\pm1$ is an arbitrary constant, A and C are given in (1) and

$$f(x,w)=\frac{R(1+R-w)^A(1+R+w)^C}{2^{A+C}(1-w)^{1+a+b}}{}_3F_2\left(\left.\begin{matrix}(a+b+1)/2,(a+b+2)/2,-x\\ a+1,-N\end{matrix}\right|-\frac{4w}{(1-w)^2}\right).$$

2. Asymptotic property:

$$c_k P_{n-k}^{(A,C)}(X)=\mathcal{O}\left(a^{n+\left[\frac{k}{3}\right]-k}\right)\quad\text{when}\quad a,\ b,\ N\to\infty\quad\text{with}\quad a\sim b\sim N.$$

4 Dual Hahn to Jacobi

1. Asymptotic expansion for large a and N:

$$\frac{(-N)_n R_n(\lambda(x); a, b, N)}{n!} = \sum_{k=0}^{n} c_k P_{n-k}^{(A,C)}(X), \quad \lambda(x) \equiv x(x+a+b+1),$$

   where $X \neq \pm 1$ is an arbitrary constant, A and C are given in (1) and

$$f(x,w) = \frac{R(1+R-w)^A(1+R+w)^C}{2^{A+C}}(1-w)^{N-x}{}_2F_1\left(\left.\begin{array}{c} -x, -b-x \\ a+1 \end{array}\right| w\right).$$

2. Asymptotic property:

$$c_k P_{n-k}^{(A,C)}(X) = \mathcal{O}\left(N^{n+\left[\frac{k}{3}\right]-k}\right) \quad \text{when} \quad a,\, N \to \infty \quad \text{with} \quad a \sim N.$$