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Optimal observation of acoustic waves

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SUMMARY

We are interested in the optimal properties of some inverse problems involving the wave equation

$$\partial_{tt} p(t,x) - \Delta p(t,x) = 0 \quad (t,x) \in [0,T] \times \Omega,$$

where p denotes an acoustic pressure propagating in a (bounded) cavity Ω and $T > 0$ is the duration of the experiment. Such a model is used for example in thermo/photo-acoustic and ultrasound tomography. A classical inverse problem is the following: *given a measurement (made by sensors) over a certain time T of the pressure p in a subdomain ω of Ω or on a part of $\partial\Omega$, can we reconstruct the initial pressure and wave velocity (at time $t = 0$) in Ω?*

The conditions guaranteeing the well-posedness of this problem are well known and imply a functional inequality called *observability inequality*. This presentation is dedicated to the following problem:

> *Is there an optimal way to position the set of sensors in order to reconstruct the initial data?*

This is a shape optimization problem in which the unknown is the domain occupied by the sensors. I will build on the series of works [1, 2, 3, 5] and will present some concrete applications and more recent results, related to the question : *can we hear the shape of a room?*

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References

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