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Analysis of a modified Euler scheme for SPDEs

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SUMMARY

A novel integrator to approximate solutions of parabolic semilinear stochastic evolution equations driven by space-time white noise

$$dX(t) = -\Lambda X(t)dt + F(X(t))dt + dW(t)$$

is presented. The proposed modified Euler scheme, introduced in [1], is written as

$$X_{n+1}^{\tau} = \mathcal{A}_{\tau}\big(X_n^{\tau} + \tau F(X_n^{\tau})\big) + \mathcal{A}_{\tau}\sqrt{\frac{\tau}{2}}\Gamma_{n,1} + \mathcal{B}_{\tau}\sqrt{\frac{\tau}{2}}\Gamma_{n,2},$$

where $\tau = T/N$, $\mathcal{A}_{\tau} = (I+\tau\Lambda)^{-1}$ $\Gamma_{n,1}, \Gamma_{n,2}$ denote independent cylindrical Gaussian random variables, and the linear operator $\mathcal{B}_{\tau}$ satisfies the condition $\mathcal{B}_{\tau}\mathcal{B}_{\tau}^{\star} = (I + \tau\Lambda)^{-1}$.

The objective of the talk is to present the main improvements when the modified Euler scheme is used instead of the standard method (given by $\mathcal{B}_{\tau} = \mathcal{A}_{\tau}$).

- The spatial regularity is preserved at all times, for any value of the time-step size τ.
- When $F = 0$, the Gaussian invariant distribution ν of the Ornstein–Uhlenbeck process $\big(X(t)\big)_{t\geq 0}$ is preserved by the numerical scheme, for any value of the time-step size τ.
- If $F = -DV$ and if an appropriate ergodicity condition is satisfied, one can approximate the invariant Gibbs distribution given by

  $$d\mu_{\star}(x) = Z^{-1}\exp(-2V(x))d\nu(x)$$

  of the process in the total variation distance: for any τ, the scheme admits a unique invariant distribution μ_{∞}^{τ}, and for all $\kappa \in (0, \frac{1}{2})$, there exists $C_{\kappa} \in (0, \infty)$ such that

  $$d_{\mathrm{TV}}(\mu_{\infty}^{\tau}, \mu_{\star}) \leq C_{\kappa}\tau^{\frac{1}{2}-\kappa}.$$

Keywords: stochastic partial differential equations, numerical approximation, invariant distribution

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References

[1] C.-E. Bréhier. Analysis of a modified Euler scheme for parabolic semilinear stochastic PDEs. *Arxiv preprint*, 2022.

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