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Optimal control of two dimensional third grade fluids

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SUMMARY

We study the optimal control of the velocity field y of an incompressible third grade fluid filling a two-dimensional bounded domain with a smooth boundary. More precisely, our aim is to solve the following problem

$$\min_{U\in\mathcal{U}_{ad}}\{\frac{1}{2}\int_0^T \|y-y_d\|_2^2 dt + \frac{\lambda}{2}\int_0^T \|U\|_2^2 dt\},$$

where the evolution equation of y is given by

$$\partial_t(v(y))-\nu\Delta y+(y\cdot\nabla)v(y)+\sum_{j=1}^{2} v(y)^j\nabla y^j-(\alpha_1+\alpha_2)div(A^2)-\beta div(|A|^2A) = -\nabla\mathbf{P}+U, \quad (1)$$

where $v(y) := y-\alpha_1\Delta y, A := A(y) = \nabla y+\nabla y^T$ and the control acts through the external force U. The constant ν represents the fluid viscosity, α_1, α_2, β are the material moduli, and $\mathbf{P}$ denotes the pressure. The equation (1) will be supplemeted with a homogeneous Navier-slip boundary condition. We recall that $y_d \in L^2(D\times(0,T))$ corresponds to a desired target field and $\lambda \geq 0$. We prove the existence of an optimal solution and establish the first order optimality conditions. Furthermore, an uniqueness result of the coupled system constituted by the state equation, the adjoint equation and the first order optimality condition is established, under sufficiently large intensity of the cost λ.

Keywords: Third grade fluids, optimal control, Necessary optimality condition.

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References

[1] Yassine Tahraoui and Fernanda Cipriano. Optimal control of two dimensional third grade fluids. *Arxiv preprint* 2022.

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