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Identification and cure of the checkerboard modes in the solution of both acoustic wave and compressible Euler systems at low Mach number flows

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SUMMARY

It is well known that Godunov type schemes applied to the numerical resolution of the compressible Euler system are not accurate at low Mach number, in particular, on Cartesian meshes. As explained in [1], this inaccuracy is occurring due to a disparity between the continuous and the discrete levels. Over the years, many fixes have been proposed to remedy these deficiencies [2]. However, fixing low Mach number accuracy problem generally introduces other issues like inaccurate acoustic computations, degraded CFL condition and inability to recover the optimal order for a Discontinuous Galerkin discretization of the Euler system [3]. Recently, a new fix was developed in [3] to avoid these problems. However, it still does not escape the checkerboard modes problem that plagues most of other low Mach number fixes.
Interestingly, there is a link between the low Mach number accuracy problem and the long time limit of a linear wave system [4]. This result is useful in explaining the low Mach accuracy problem encountered in the discrete compressible Euler system through an analysis of the linear wave system. In this work, we first propose to exploit this link to identify the origin of the checkerboard modes in the solution of the Euler system. A discrete Hodge-Helmholtz decomposition of the velocity field for the wave system is used to identify the term responsible for the appearance of the checkerboard modes. Next, we investigate the utility of the filtering method towards removing the checkerboard mode from the numerical solution of the linear wave system and eventually the Euler system.

Keywords: Finite volume methods, low Mach flows, checkerboard modes.

AMS Classification: 65M08, 65N22, 76N99

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