

Implicit and semi-implicit high-order well-balanced finite-volume methods for general 1D systems of balance laws

Irene Gómez-Bueno¹, S. Boscarino², M.J. Castro³, C. Parés³ G. Russo²

SUMMARY

The aim of this work is to design implicit and semi-implicit high-order well-balanced numerical methods for general one-dimensional systems of balance laws. The technique introduced by two of the authors in [1] for explicit methods is considered, whose key is the application of well-balanced reconstruction operators. The well-balanced property is preserved when quadrature formulas are used to approximate the cell averages and the integrals of the source term in the cells. This procedure is combined with a time discretization method for the time evolution of type RK-IMEX or RK-implicit (see [2]). The methodology will be applied to several systems of balance laws, ranging from simple scalar problems such as the Burgers equation to more complex systems like the shallow water equations with Manning friction.

Keywords: PDEs, systems of balance laws, finite-volume methods, high-order methods, implicit methods, IMEX methods, well-balanced methods. . .

References

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¹University of Málaga, Spain.
email: igomezbueno@uma.es

²University of Málaga, Spain.

³University of Málaga, Spain.