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A Time–Dependent Generalisation of a Radzievskij Problem

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SUMMARY

V. V. Radzievskij [2] tackled the problem of motion of two point masses inside a homogeneous, rarefied spherical cloud, assuming that the mutual interactions between these point bodies and between the bodies and the particles of the cloud are described by their mutual Newtonian gravitational attraction. As an additional, symplifying hypothesis, the constant density of the cloud is supposed to be sufficiently small so that the resistance of the surrounding material medium to the motion of those point masses might be neglected.

Under these assumptions, *the problem of relative motion of these two bodies can be reformulated as a perturbed Keplerian system, in which the perturbing effects are due to a conservative central force.* Taking advantage of the first integrals of the angular momentum and the energy, Radzievskij followed the conventional solution procedure in plane polar coordinates (r, φ) within the (invariant) plane of motion, formally leading to an orbit equation (in inverted form), $\varphi = \varphi(r; r_0, \varphi_0)$, in terms of *an Abelian integral.* Note that, although this author states that the problem can be analytically solved in terms of Abelian integrals, the fact is that the required quadratures can be reduced to *elliptic integrals.*

In this talk we consider analytical approaches to the solution of *a time–dependent generalisation of the said Radzievskij problem* that can be formulated as the study of motion of a unit–mass particle within a certain time–dependent central–force field.

In particular, the generalized Radzievskij problem under consideration can be viewed as *a perturbed Gyldén system* (that is, a Keplerian system with a time–varying Keplerian coupling parameter $\mu(t)$) *in which the perturbing force is also a time–dependent central force.* This problem does not possesses the classical first integral of the total energy of the system any more, but the force model can be derived from a time–dependent scalar potential.

To deal with this time–dependent central–force problem we take our cue from Deprit ([1], §3, pp. 7–10) and identify the elliptic integrals required to develop the transformation.

Keywords: Celestial Mechanics, Radzievskij's three–body problem, perturbed Gyldén systems, time–dependent central force, canonical transformations, generating functions.

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References

[1] Deprit, A. The Secular Accelerations in Gylden's Problem. *Celestial Mechanics* **31**(1), 1–22, 1983.

[2] Radzievskij, V. V. General Solution of a Case of the Problem of Three Bodies. *Transactions (Doklady) of the USSR Academy of Sciences* (New Series) **91**(6), 1309–1311, 1953 (in Russian).

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