

Two-point Taylor expansions in singular one-dimensional boundary value problems: Application to the spheroidal wave equation

Chelo Ferreira¹, José L. López², Ester Pérez Sinusía¹

SUMMARY

We consider the second-order linear differential equation $(x^2 - 1)y'' + f(x)y' + g(x)y = h(x)$ in the interval $(-1, 1)$ with initial conditions or boundary conditions (Dirichlet, Neumann or mixed Dirichlet-Neumann). The functions $f(x)$, $g(x)$ and $h(x)$ are analytic in a Cassini disk \mathcal{D}_r with foci at $x = \pm 1$ containing the interval $[-1, 1]$. Then, the two end points of the interval may be regular singular points of the differential equation. The two-point Taylor expansion of the solution $y(x)$ at the end points ± 1 is used to study the space of analytic solutions in \mathcal{D}_r of the differential equation, and to give a criterion for the existence and uniqueness of analytic solutions of the boundary value problem. The method is constructive and provides the two-point Taylor approximation of the analytic solutions when they exist. We apply it in the study of eigenfunctions of the spheroidal wave equation.

Keywords: second-order linear differential equations, regular singular point, Frobenius method, two-point Taylor expansions, spheroidal wave equation.

AMS Classification: 34A25, 34B05, 41A58

References

- [1] C. Ferreira, J. L. López and E. Pérez Sinusía, The use of two-point Taylor expansions in singular one-dimensional boundary value problems I, *J. Math. Anal. Appl.*, **463**(2), 708–725, 2018.
- [2] J. L. López, E. Pérez Sinusía and N. Temme, Multi-point Taylor approximations in one-dimensional linear boundary value problems, *Appl. Math. Comput.*, **207**, 519–527, 2009.

¹Departamento de Matemática Aplicada
Universidad de Zaragoza, IUMA
50009 Zaragoza
cferrei@unizar.es, ester.perez@unizar.es

²Departamento de Estadística, Informática y Matemáticas, INAMAT
Universidad Pública de Navarra
31006 Pamplona
jl.lopez@unavarra.es