Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 10–12th 2018

Uniformly convergent expansions of the Generalized Hypergeometric function ${}_pF_q$ in terms of elementary functions.

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SUMMARY

The solution of the following linear equation of order $q+1$

$$\left[z\frac{d}{dz}\prod_{j=1}^{q}\left(\frac{d}{dz}+b_j-1\right) - z\prod_{i=1}^{p}\left(z\,\frac{d}{dz}+a_i\right)\right] w = 0,$$

is the generalized hypergeometric function ${}_pF_q(a_1,...,a_p\,,\,b_1,...,b_q\,;\,z)$ with $p+q$ parameters. This function is useful in statistic and also appear in the evaluation of the so-called Watson integrals which characterize the simplest possible lattice walks. It's also potentially useful for the solution of more complicated restricted lattice walk problems.

In general, the generalized hypergeometric function ${}_pF_q$ has not an explicit expresion and to compute it, we have to use approximation techniques. To this end, the most used are the Taylor or asymptotic expansion, valid for small and large values of the argument z respectively.

In this work, we derive new expansions of the generalized hypergeometric function ${}_pF_q$ in terms of elementary functions of z that converge in different regions, bounded or unbounded, of the complex plane. We give either, explicit formulas for the coefficients of the expansions. Using a integral representation of of ${}_pF_q$, the key point of the analysis is the approximation of an appropriate factor of the integrand.

Finally, we show the accuracy of the approximations by means of several numerical experiments.

Keywords: generalized hypergeometric function, convergent expansions

AMS Classification: 33C20, 41A58, 41A80

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