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Development of a control tool for releases of pollutants in rivers

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SUMMARY

The objective of this work is to control the environmental impact produced by point and non-point sources discharged to natural bodies of water. The optimal control problem for a pollutant in 1D is based on a gradient-based method. This technique involves to solve the convection-diffusion equation of the transport of a pollutant in rivers and streams [1]

$$\frac{\partial \phi}{\partial t} + \frac{\partial (u\phi)}{\partial x} - E\frac{\partial}{\partial x}\left(\frac{\partial (u\phi)}{\partial x}\right) = f(x,t) \tag{1}$$

where f is a a controllable source to adjust the concentration (ϕ) to a given objective function defined at a target location J, and u the flow velocity. And its adjoint equation:

$$-\frac{\partial \sigma}{\partial t} - \frac{\partial (u\sigma)}{\partial x} - E\frac{\partial}{\partial x}\left(\frac{\partial (u\sigma)}{\partial x}\right) = (\phi - \phi_{obj}) \tag{2}$$

This system of equations is solved by means of an explicit first order upwind scheme, considering backward temporal-integration for the adjoint equation to minimize the error. The gradient descent method can be formulated as:

$$f^{k+1} = f^k + \varepsilon \nabla J^k \tag{3}$$

In order obtain global convergence, not only the step lengths must be chosen properly, but also the search direction. Such that $J^{k+1} < J^k$. The sensitivity of J with respect to the perturbations in the source terms f can be expressed as:

$$\nabla J(x,t) = \frac{\delta J}{\delta f(x,t)} = -\sigma(x,t) \tag{4}$$

The term σ is the sensitivy vector, which is used to adjust the concentration at any other point.

Keywords: Pollutant, optimization, gradient method

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References

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