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Laplacian (co)flow of a locally conformal parallel G_2-structure

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SUMMARY

The development of flows in Riemannian geometry has been mainly motivated by the study of the Ricci flow. However, the same techniques are also useful in the study of flows involving other geometrical structures, like for example, the Kähler Ricci flow. Concerning flows on G_2-manifolds, for any closed G_2-structure σ_0 on a manifold M, Bryant (Proceedings of Gökova Geometry-Topology Conference 2005, 2006) introduced a natural flow, the so-called *Laplacian flow*, given by

$$\begin{cases} \frac{d}{dt}\sigma(t) = \Delta_t\sigma(t), \\ \sigma(0) = \sigma_0, \quad d\sigma(t) = 0, \end{cases}$$

where Δ_t is the Hodge Laplacian operator of the metric determined by $\sigma(t)$. The short time existence and uniqueness of solution for the Laplacian flow of any closed G_2-structure, on a compact manifold M, has been proved by Bryant and Xu in the unpublished paper arxiv:1101.2004[math.DG].

Karigiannis, McKay and Tsui (Diff. Geom. Appl. 2012) introduced the *Laplacian coflow*. In this case the initial G_2-form is claimed to be coclosed, i.e. $d*\sigma_0 = 0$. Up to now, short time existence of solution of the coflow is not known. Assuming short time existence and uniqueness of solution, the authors show that the condition of the initial G_2-form σ_0 to be coclosed (equiv. ψ_0 closed) is preserved along the flow.

Here we are concerned with studying the Laplacian flow, resp. coflow, of an LCP G_2-structure on a manifold M defined as:

$$\begin{cases} \frac{d}{dt}\sigma(t) = \Delta_t\sigma(t), \\ \sigma(0) = \sigma_0, \\ d\sigma(t) = 3\,\tau(t)\wedge\sigma(t), \\ d*_t\sigma(t) = 4\,\tau(t)\wedge *_t\sigma(t). \end{cases} \qquad \begin{cases} \frac{d}{dt}\psi(t) = -\Delta_t\psi(t), \\ \psi(0) = \psi_0, \\ d\psi(t) = 4\,\tau(t)\wedge\psi(t), \\ d*_t\psi(t) = 3\,\tau(t)\wedge *_t\psi(t). \end{cases}$$

The first examples of long time solutions of these flows are given. Our examples are one-parameter families of Locally Conformal Parallel G_2-structures on solvable Lie groups. We start finding solutions for the Laplacian flow and the found solutions are used to construct long time solutions to the Laplacian coflow starting from a Locally Conformal Parallel structure. These results can be found in the preprint available in arxiv:1711.08644[math.DG].

Keywords: Geometric flows, G_2-structures, locally conformal parallel structures,...

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