

Fractal approximation for 3D periodic data

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SUMMARY

We study a trigonometric type fitting map for functions or data defined on a grid on the two-dimensional interval $[-\pi, \pi] \times [-\pi, \pi]$, assuming periodicity in both variables. Let us consider a set of three-dimensional data on a grid, for $f \in \mathcal{C}(T^1 \times T^1)$ ([1], [2], [3]):

$$\{(x_i, y_j, f(x_i, y_j)) : i = 1, 2, \dots, m; j = 1, 2, \dots, n\},$$

where $x_{i+1} - x_i = \pi/m$, $i = 1, 2, \dots, 2m - 1$; $y_{j+1} - y_j = \pi/n$; $j = 1, 2, \dots, 2n - 1$, and

$$\mathcal{J}_{mn\gamma}(f)(x, y) = K_{mn\gamma}(x, y) \sum_{i=1}^{2m} \sum_{j=1}^{2n} f(x_i, y_j) \left| \frac{\sin(\frac{1}{2}m(x_i - x))}{m \sin(\frac{1}{2}(x_i - x))} \right|^\gamma \left| \frac{\sin(\frac{1}{2}n(y_j - y))}{n \sin(\frac{1}{2}(y_j - y))} \right|^\gamma, \quad (1)$$

where

$$K_{mn\gamma}^{-1}(x, y) = \sum_{i=1}^{2m} \sum_{j=1}^{2n} \left| \frac{\sin(\frac{1}{2}m(x_i - x))}{m \sin(\frac{1}{2}(x_i - x))} \right|^\gamma \left| \frac{\sin(\frac{1}{2}n(y_j - y))}{n \sin(\frac{1}{2}(y_j - y))} \right|^\gamma. \quad (2)$$

These approximants are extended to systems of fractal functions which are smooth or non-smooth depending on the choice of scaling factors. We obtain bounds of the approximation error and the convergence of the fitting functions with very weak conditions, when the sampling frequency is increased. The density of these mappings in the space of two-dimensional periodic and continuous functions is proved.

Keywords: Fractal Interpolation Functions, Two-dimensional Approximation, Fractals

AMS Classification: 28A80, 42A10, 42A15

References

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