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Multidimensional discrete PDE splines using radial basis functions

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SUMMARY

Radial basis function (RBF) methods have emerged as an important and effective tool for the numerical solution of partial differential equations (PDE) in any number of dimensions and for the approximation of an unknown multivariate function by interpolation at scattered sites [2, 4], entering in a field traditionally tackled by finite element methods (FEM) [3].

Also, PDE surfaces, which are surfaces whose behaviour is governed by PDEs [1], have been shown to possess many modelling advantages in a wide range of fields. A combination of conditions of interpolation and approximation can be used for the PDE method of surface design: on one hand, the surface has to approximate a given data set, and on the other hand, it has to be modelled by a partial differential equation. In addition, the surface has to satisfy some boundary conditions that are included along with the equation as a boundary value problem. Moreover, this 2-dimensional approximation problem may be generalized to the d-dimensional case, for any positive integer d.

By using RBF techniques we study the existence and the uniqueness of the solution of the generalized problem in a Lipschitz domain and arbitrary dimension. We formulate our variational problem in an adequate function space, the native space, and we discretize the solution in terms of RBF. Moreover, we show convergence and derive error estimates.

Keywords: Approximation, interpolation, radial basis functions, PDE

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