

## Optimization methods for computing periodic orbits

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### SUMMARY

An autonomous dynamical system,  $\dot{\mathbf{x}} = F(\mathbf{x})$ , has a periodic orbit if there is a vector  $\mathbf{x}_0$  and a scalar  $T$  such that  $\|\mathbf{x}(T; \mathbf{x}_0) - \mathbf{x}_0\| = 0$ . Therefore, the problem of finding periodic orbits may be considered from a new view point, that is, to find the zeros of a non-negative  $f : \mathcal{D} \subset \mathbb{R}^n \rightarrow \mathbb{R}^+$ , or equivalently, since the function is non-negative, to find the absolute minima of the function  $f$  in the domain  $\mathcal{D}$ .

Modern techniques of evolutionary computation, like genetic algorithms or evolution strategies, inspired on biological processes, are currently used in different scientific areas to solve problems that involve optimization of functions [2]. As we just said, we may consider the problem of finding periodic orbits as a problem of optimization, and hence this kind of methods may be applied to the determination of periodic orbits.

The problem of finding periodic orbits has no unique solution (let us remind that periodic orbits can be dense in the phase space). In order to avoid accumulation of solutions around a particular point while abandoning other regions with solutions, a modification of the Evolution Strategy Method (that find only one solution), based on [3], can be used [1].

**Keywords:** periodic orbits, evolution strategies

**AMS Classification:** 37C27, 65K10, 90C59

### References

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