Fifteenth International Conference
Zaragoza-Pau
on Mathematics and its Applications

Abstracts

Jaca, September 10–12th 2018

Talks and Posters

Contributors

Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Optimization methods for computing periodic orbits

Alberto Abad

SUMMARY

An autonomous dynamical system, $\dot{\boldsymbol{x}} = F(\boldsymbol{x})$, has a periodic orbit if there is a vector $\boldsymbol{x}_0$ and a scalar T such that $\|\boldsymbol{x}(T;\boldsymbol{x}_0) - \boldsymbol{x}_0\| = 0$. Therefore, the problem of finding periodic orbits may be considered from a new view point, that is, to find the zeros of a non-negative $f : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{R}^+$, or equivalently, since the function is non-negative, to find the absolute minima of the function f in the domain $\mathcal{D}$.

Modern techniques of evolutionary computation, like genetic algorithms or evolution strategies, inspired on biological processes, are currently used in different scientific areas to solve problems that involve optimization of functions [2]. As we just said, we may consider the problem of finding periodic orbits as a problem of optimization, and hence this kind of methods may be applied to the determination of periodic orbits.

The problem of finding periodic orbits has no unique solution (let us remind that periodic orbits can be dense in the phase space). In order to avoid accumulation of solutions around a particular point while abandoning other regions with solutions, a modification of the Evolution Strategy Method (that find only one solution), based on [3], can be used [1].

Keywords: periodic orbits, evolution strategies

AMS Classification: 37C27, 65K10, 90C59

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A finite volume method for fully coupled multiphase flow and chemical processes in porous media-application to CO_2 storage

Etienne Ahusborde[1], Mustapha El Ossmani[1,2], Mohamed Id Moulay[1]

SUMMARY

Reactive multiphase flows play a significant role in many applications related to environmental sciences or petroleum engineering. We can mention, non exhaustively, geological sequestration of CO2 in deep saline aquifers, long term management of nuclear waste or enhanced oil recovery, see for instance [1].

Modelling such problem leads to a highly nonlinear coupled system of PDEs (governing the compositional flow) to algebraic or ordinary differential equations (governing respectively equilibrium and kinetic reactions) requiring a special numerical treatment. In the framework of the free and open source simulator DuMu$^{\mathrm{X}}$ (http://www.dumux.org/), we have developed and implemented a fully-implicit finite volume scheme for the numerical simulation of single and two-phase reactive flows.

In this talk, we will describe in detail the methodology for single phase reactive flows where a direct substitution approach (DSA) is employed. The accuracy and effectiveness of the approach have been demonstrated through 2D and 3D parallel numerical simulations. Numerical results for long-term fate of injected CO_2 for geological sequestration will be presented. Moreover, benefits of the DSA will be highlighted by a comparison with results obtained by a sequential iterative approach (SIA) developed in [2]. The extension to two-phase flows will be discussed and some preliminary numerical results will be presented.

Acknowledgments

This work was partially supported by the Carnot Institute, ISIFoR project (Institute for the Sustainable Engineering of Fossil Resources). This support is gratefully acknowledged. We also thank the DuMu$^{\mathrm{X}}$ and DUNE teams for their help during the development of our reactive transport module and the CINES (National Computing Center for Higher Education) to give us access to their computing resources facility under the allocation 2018-A0040610019 made by GENCI.

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Walls in a Junction of three Ferromagnetic Nanowires

Abdel kader Al SAYED, **Gilles Carbou**[1], **Stéphane Labbé**[2],

SUMMARY

This work is devoted to study the effect of a junction of ferromagnetic wires on the walls profile. More specifically, we focus on the effect of the junction of three wires when we connect perpendicularly a finite straight wire on a straight infinite horizontal wire. The variation of the magnetization satisfy the Landau Lifschitz equation coupled by conditions on the junction and the homogeneous Neumann boundary condition. We exhibit all the construction of the static solutions describing either one or two domains separated by a domain wall. We address the stability of these solutions using mobile frame technique to take into account only the perturbations which verify the physical constraint having norm is equal to one (see[2]). We also perform numerical simulations using Python with an explicit scheme in order to illustrate the obtained stability results.

Keywords: ferromagnetism, Landau-Lifschitz equation, stability, domain walls....

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Fractional Powers of the Stokes operator with boundaryconditions involving the pressure

Hind Al Baba

SUMMARY

Stokes and Navier-Stokes problems have been often studied with Dirichlet boundary condition. Nevertheless, in the opinion of engineers and physicists such a condition is not alwaysrealistic in industrial and applied problems of origin. Thus arises naturally the need to carryout a mathematical analysis of these systems with different boundary conditions, which bestrepresent the underlying fluid dynamic phenomenology. Based on the study of the complexand fractional powers of the Stokes operator with pressure boundary condition, we carry outa systematic treatment of the Stokes problem with the corresponding boundary conditionsin Lp-spaces.

Keywords: Stokes Problem, pressure boundary conditions, Complex and fractional powers of operators.

AMS Classification: F35B65, 35D30, 35D35, 35K20, 35Q30, 76D05, 76D07, 76N10

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Semigroup theory for the Stokes operator with Navier boundary condition on L^p spaces

C. Amrouche[1], M. Escobedo[1], A. Ghosh[1,2]

SUMMARY

SUMMARYWe consider the motion of a viscous incompressible fluid given by non-stationary Navier- Stokes equation with slip boundary condition in a bounded domain

$$\frac{\partial u}{\partial t}?\nabla u + (u \cdot \nabla)u + \nabla\pi = 0, \div u = 0 \text{in } \Omega \times (0,T);$$

$$u \cdot n = 0, 2[(Du)n]_\tau + \alpha u_\tau = 0 \text{on } \Lambda \times (0,T); (1)?$$

$$u(0) = u_0 \text{in } \Omega.$$

Here Ω is a bounded domain in R^3 with boundary Λ. The initial velocity u_0 and the (scalar)friction coefficient α are given functions; The external unit normal vector on Λ is denoted by$n$, $Du = 1/2(\nabla u + \nabla^T u$ denotes the strain tensor and the subscript τ denotes the tangential component i.e. $v_\tau = v?(v \cdot n)n$ for any vector field v. The functions u and π describe respectively the velocity and the pressure of the fluid.The boundary condition in (1) was introduced by H. Navier (in [1]) which is in recent years widely studied because of its significance in real world in different model for simulation of flows and fluid-solid interaction problems (cf. [2]).The well-posedness of the above system imposing minimal regularity on α will be discussed. We use semigroup theory to first study the weak and strong solutions for the associated Stokes operator. Resolvent estimate uniform with respect to α is deduced which enables us to have bounds on the solution u of (1) independent of α. Finally we study the behaviour of the solution of (1) with respect to the friction coefficient, in particular what happens if α goes to ∞.

Keywords: Navier-Stokes equation, slip boundary condition, semigroup theory, dependence on friction coefficient

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Elliptic Problems in Smooth and Non Smooth Domains

Chérif Amrouche[1] Mohand Moussaoui[2] Huy Hoang Nguyen [1,3]

SUMMARY

We are interested here in questions related to the regularity of solutions of elliptic problemswith Dirichlet or Neumann boundary condition (see ([1]). For the last 20 years, lots ofwork has been concerned with questions when is a Lipschitz domain.We give here some complements for the case of the Laplacian (see [3]), the Bilaplacian([2],[6]) and the operator div (A∇) (see ([5]), when A is a matrix or a function, and we extendthis study to obtain other regularity results for domains having an adequate regularity.Using the duality method, we will then revisit the work of Lions-Magenes [4], concerningthe so-called very weak solutions, when the data are less regular. Thanks to the interpolation theory, it permits us to extend the classes of solutions and then to obtain newresults of regularity.

Keywords: Elliptic problems, Lipschitz domains, regularity

AMS Classification: 35C15, 35J25, 35J40

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Triangular curves

Enrique Artal[1]

SUMMARY

In this talk, given d at least three we present Zariski tuples of curves parametrized by the d-roots of unity (up to conjugation). Moreover, these examples include arithmetic Zariski tuples associated with any cyclotomic polynomial of d-roots of unity. If d is at least 4, the fundamental groups of the complements are always abelian. Some open questions will be presented. It is a joint work with J.I. Cogolludo and J. Martín-Morales.

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Multiplicative Lie triple higher derivations on generalized matrix algebras

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Abstract

Let $\mathbb{N}$ be the set of nonnegative integers and $\mathcal{G} = (\mathcal{A}, \mathcal{M}, \mathcal{N}, \mathcal{B})$ be a 2-torsion free generalized matrix algebra over a commutative ring $\mathcal{R}$. In the present paper, under some lenient assumptions on $\mathcal{G}$, it is shown that if $\Delta = \{\delta_n\}_{n\in\mathbb{N}}$ is a sequence of mappings $\delta_n : \mathcal{G} \to \mathcal{G}$ (not necessarily linear) satisfying $\delta_n([[a,b],c]) = \sum_{r+s+t=n} [[\delta_r(a), \delta_s(b)], \delta_t(c)]$ for all $a, b, c \in \mathcal{G}$, then for each $n \in \mathbb{N}$, $\delta_n = d_n + \tau_n$; where $d_n : \mathcal{G} \to \mathcal{G}$ is an additive mapping satisfying $d_n(ab) = \sum_{r+s=n} d_r(a)d_s(b)$ for all $a, b \in \mathcal{G}$, i.e., $\mathcal{D} = \{d_n\}_{n\in\mathbb{N}}$ is an additive higher derivation on $\mathcal{G}$ and $\tau_n : \mathcal{G} \to Z(\mathcal{G})$(where $Z(\mathcal{G})$ is the center of $\mathcal{G}$) is a map vanishing at every second commutator $[[a,b],c]$.

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Towards extending the W, Z paradigm for first passage problems of Lévy processes to strong Markov processes with one sided jumps

Florin Avram

SUMMARY

The 'paradigm' alluded in the title is a conjecture, which states that the tens of W, Z formulas [1] for the first passage problems of Lévy processes with one sided jumps apply equally to strong Markov processes with one sided jumps. It has been largely established already for Markov additive processes [5], for Lévy processes with Ω state dependent killing [7] and for random walks [3].Recent works of [6], [4], [2] show it may apply to strong Markov processes with general draw-down stopping (which generalizes classic and draw-down /trailing stop).Applications involving exit and optimizing dividends will be presented.

Keywords: exit problem, spectrally one sided process, draw-down process, scale functions, dividends

AMS Classification: 60H10, 60J80, 60J25

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Optimal Replacement Policy under a General Failureand Repair Model: Minimal versus Worse Than OldRepair

Badía, F.G.[1], Berrade, M.D.[1], Cha, J.H.[2], Lee, H.[2]

SUMMARY

We analyze the optimal replacement policy for a system subject to a general failure andrepair model. Failures can be of one of two types: catastrophic or minor. The formerleads to the replacement of the system, whereas minor failures are followed by repairs. Thenovelty of the proposed model is that, after repair, the system recovers the operational statebut its condition is worse than that just prior to failure (worse than old). Undertrainedoperators or low quality spare parts explain this deficient maintenance. The correspondingfailure process is based on the Generalized Polya Process which presents both the minimalrepair and the perfect repair as special cases. The system is replaced by a new one afterthe first catastrophic failure, and also undergoes two sorts of preventive maintenance basedon age and after a predetermined number of minor failures whichever comes first. Wederive the long-run average cost rate and study the optimal replacement policy. Somenumerical examples illustrate the comparison between the as bad-as-old and the worse thanold conditions.

Keywords: Maintenance, Generalized Polya process, worse-than-minimal-repair, optimum policy

AMS Classification: 90B25, 60K10

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Stochastic comparisons and multivariate dependence for the epoch times of trend renewal processes

F.G. Badía[1], **C. Sangüesa**[2], **J.H. Cha**[3]

SUMMARY

We study stochastic comparisons and dependence properties for the epoch and inter epoch times of trend renewal processes with (possibly) different baseline renewal process and trend functions. These results extend some of the results on the nonhomogeneous Poisson process in Belzunce et al. [1, 2] to those on trend renewal process. Some applications of the obtained results to a shock model, a repair process and some specific class of intermediate order statistics are provided.

Keywords: Stochastic comparison; trend renewal process; epoch times; dependence properties

AMS Classification: 60E15, 60K05, 60K10

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Matroids, two-graphs and the embedded topology of quartics and bitangent lines

Shinzo Bannai[1]

SUMMARY

Smooth quartic curves and their bitangent lines are very classical and interesting objects studied by many mathematicians. In this talk, concepts from graph theory and combinatorics, namely matroids and two-graphs are introduced and applied to the study of the embedded topology of arrangements consisting of smooth quartics and bitangent lines, providing another interesting aspect to the study of quartic curves.

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A phytoplankton aggregation study by the spatial moments approximation of Individual-based Model

Naziha Bordj, Nadjia El Saadi[1].

SUMMARY

The aim of this work is to study the phenomenon of aggregation in a phytoplankton population by dynamics system of spatial moments corresponding to integro-differential equations allowing to analyse population dynamics and its spatial structure. This model is developed from a phytoplankton Individual-Based Model (IBM), that is built on the basis of stochastic processes describing the branching (cell division or death) under the effect of a local competition in the division; and the movement taking into account the random dispersion and the spatial interactions between cells due to their chemosensory abilities.

Keywords: Phytoplankton, Aggregation, Competition, Dispersion, Individual-Based Model (IBM), Stochastic process, Spatial Moments, Closure moment, spatial dynamics, Integro-differential equation, Spatial and time discretization

AMS Classification: 92D25, 97N40, 45G15

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Quasi-Interpolation and Applications to PDEs with Radial Basis Functions

Martin Buhmann

SUMMARY

Quasi-interpolation and interpolation with radial basis functions are the most often used methods of approximation in multiple space dimensions by shifts of kernel functions. The advantages of quasi-interpolation are manifold: they are suitable for smoothing for instance and allow function information not only to be provided by point-wise evaluation, but also by local integrals, divided differences etc. In this talk we shall speak about quasi-interpolation and convergence orders using shifts of radial basis functions, and we shall also mention a new method to solve partial differential equations with radial basis functions.

(Joint work with Joaquin Jodar/Jaén University, and with Miguel Rodríguez/Granada University.)

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Multidimensional discrete PDE splines using radial basis functions

M.D. Buhmann[1], J. Jódar[2], M.L. Rodríguez[3]

SUMMARY

Radial basis function (RBF) methods have emerged as an important and effective tool for the numerical solution of partial differential equations (PDE) in any number of dimensions and for the approximation of an unknown multivariate function by interpolation at scattered sites [2, 4], entering in a field traditionally tackled by finite element methods (FEM) [3].

Also, PDE surfaces, which are surfaces whose behaviour is governed by PDEs [1], have been shown to possess many modelling advantages in a wide range of fields. A combination of conditions of interpolation and approximation can be used for the PDE method of surface design: on one hand, the surface has to approximate a given data set, and on the other hand, it has to be modelled by a partial differential equation. In addition, the surface has to satisfy some boundary conditions that are included along with the equation as a boundary value problem. Moreover, this 2-dimensional approximation problem may be generalized to the d-dimensional case, for any positive integer d.

By using RBF techniques we study the existence and the uniqueness of the solution of the generalized problem in a Lipschitz domain and arbitrary dimension. We formulate our variational problem in an adequate function space, the native space, and we discretize the solution in terms of RBF. Moreover, we show convergence and derive error estimates.

Keywords: Approximation, interpolation, radial basis functions, PDE

AMS Classification: 41A30, 41A63, 65N15

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An evolutionary algorithm for the Multi-Period Facility Location Problem

Herminia I. Calvete[1], Pedro M. Mateo[2], Carlos Sández

SUMMARY

Facility Location problems study the optimal allocation of a set of facilities so as to supply some demand nodes while minimizing the allocation and maintenance costs. Much of the available literature on facility location is focused on single-period assignments, and this is unrealistic for many use cases.

A model belonging to the class of facility location problems which addresses those issues is the multi-period incremental service facility location problem (MISFLP). The MISFLP was first presented in [1] and considers a multi-period setting where the minimum number of facilities to open is fixed for each period. This allows for a more gradual deployment of a service, suitable for private sectors and non-essential necessities.

The goal of this study is to devise the first heuristic procedure to obtain optimal or near-optimal solutions for the MISFLP through the use of a specially tailored evolutionary algorithm. A computational study is carried out to assess the performance of the algorithm.

Keywords: Evolutionary algorithm, Multi-period, Location, MISFLP

AMS Classification: 90B80, 90C59, 90C05

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A free energy diminishing DDFV scheme for convection-diffusion equations

Clément Cancès, Claire Chainais-Hillairet[1], Stella Krell[2],

SUMMARY

In my talk, I will introduce a nonlinear Discrete Duality Finite Volume scheme to approximate the solutions of drift diffusion equations. The scheme is built to preserve at the discrete level even on severely distorted meshes the energy / energy dissipation relation. This relation is of paramount importance to capture the long-time behavior of the problem in an accurate way. To enforce it, the linear convection diffusion equation is rewritten in a nonlinear form before being discretized. This is a joint work with Clément Cancès (Lille) and Stella Krell (Nice).

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Walker Regime for Walls in Ferromagnetic Nanotubes

Gilles Carbou[1]

SUMMARY

Ferromagnetic nanotubes are proposed in [1] as an alternative to ferromagnetic nanowires for data-storage applications. We consider a two-dimensional model for such devices and we establish the stability of moving walls in the Walker regime when the tube is subject to a small magnetic field.

Keywords: Landau-Lifschitz equation, domain walls, stability

AMS Classification: 35K55, 35Q60

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 10–12th 2018

A higher-order method on a graded mesh for a time-fractional diffusion problem

Hu Chen, Martin Stynes[1]

SUMMARY

Alikhanov's high-order scheme for Caputo fractional derivatives of order $\alpha \in (0,1)$ is generalised to nonuniform meshes and analysed for initial-value problems (IVPs) and initial-boundary value problems (IBVPs) whose solutions display a typical weak singularity at the initial time. It is shown that, when the mesh is chosen suitably, the scheme attains order $3-\alpha$ convergence for the 1-dimensional IVP and second-order convergence for the IBVP. For the IBVP we consider the case where the spatial domain is the unit square and use a spectral method to discretise in space, but other spatial domains and other spatial dimensions and discretisations are possible. Numerical results demonstrate the sharpness of the theoretical convergence estimates.

The talk is aimed at non-specialists but it does assume some familiarity with finite difference methods.

Keywords: Caputo fractional derivative, Alikhanov scheme

AMS Classification: 65M06, 65M70

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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An efficient uniformly convergent method for solving singularly perturbed semilinear reaction-diffusion systems

C. Clavero, J.C. Jorge[1],

SUMMARY

In this talk we develop and analyze a numerical method for solving 1D semilinear parabolic singularly perturbed systems of reaction-diffusion type, which are given by

$$\begin{cases} \dfrac{\partial \mathbf{u}}{\partial t}(x,t) - \mathcal{D}_\varepsilon \dfrac{\partial^2 \mathbf{u}}{\partial x^2}(x,t) + \mathcal{A}(x,t,\mathbf{u}) = \mathbf{0}, \ (x,t) \in (0,1)\times(0,T], \\ \mathbf{u}(0,t) = \mathbf{g_1}(t), \ \mathbf{u}(1,t) = \mathbf{g_2}(t), \ \forall\, t \in (0,T], \ \mathbf{u}(x,0) = \varphi(x), \ \forall\, x \in [0,1], \end{cases}$$

being $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$, $\mathcal{D}_\varepsilon =$ diag $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ satisfying $0 < \varepsilon_1 \leq \varepsilon_2 \leq \ldots \leq \varepsilon_n \leq 1$ and the nonlinear reaction term $\mathcal{A}(x,t,\mathbf{u}) = (a_1(x,t,\mathbf{u}), a_2(x,t,\mathbf{u}), \ldots, a_n(x,t,\mathbf{u}))^T$ is composed by sufficiently smooth functions a_i such that, for all $(x,t.\mathbf{v}) \in [0,1]\times[0,T]\times\mathcal{R}^n$, it holds that

$$\frac{\partial a_i}{\partial v_j}(x,t,\mathbf{v}) \leq 0, \ i \neq j, \ i,j = 1,\ldots,n,$$
$$\sum_{j=1}^{n} \frac{\partial a_i}{\partial v_j}(x,t,\mathbf{v}) \geq \alpha > 0, \ i = 1,\ldots,n.$$

In the case of having small diffusion parameters ε_i with different orders of magnitude overlapping boundary layers appear close to the end points of the interval $(0,1)$.

The numerical method which we propose combines a linearized version of the fractional implicit Euler method together with a splitting by components, to discretize in time, and the central finite difference scheme on an appropriate piecewise uniform mesh, to discretize in space. In this way, only small tridiagonal linear systems are involved in the numerical integration in time.

It is proven and tested that the proposed numerical algorithm is uniformly convergent, of first order in time and of almost second order in space.

Keywords: semilinear parabolic systems, reaction-diffusion, splitting, Shishkin meshes, uniform convergence.

AMS Classification: 65N05, 65N06, 65N10

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Modeling with fractional derivatives - spread of the dengue fever epidemic

Jacky Cresson, Anna Szafrańska[1]

SUMMARY

The dynamic behavior of epidemic disease is an important topic in the real world. This problem was studied in the literature from a very long time and plenty of mathematical researchers had tried to describe this phenomenon using mathematical tools. An important branch of this research is modeling using systems of ordinary differential equations or partial differential equations. Nevertheless, it appears that in some cases, like for the dengue fever epidemic, this framework seems insufficient to capture some part of the dynamics. This is in particular the case when some memory effects occur. One way to take into account such kind of behavior is to change the setting of ordinary differential equations to fractional differential equations. This change can be in some cases interpreted and the relation between the classical and fractional modeling can be precised. In this talk, we will present a global view of such a framework with numerous concrete examples. A special emphasis is given to the modeling problem of the spread of the dengue fever epidemic. We construct a new fractional model and we provide supporting numerical simulations for the validity of this model.

Keywords: Fractional calculus, Dengue fever, embedding formalism,...

AMS Classification: 26A33, 49M25, 65Q30

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Abstract fractional differential equations with order varying in time in complex Banach spaces and its time discretization: Well–posedness, regularity, and asymptotic behavior

E. Cuesta[1], **R. Ponce**[2],

SUMMARY

An abstract time fractional differential equation with order varying in time

$$u(t) = u_0 + \partial_t^{-\beta(t)} Au(t) + f(t), \qquad t > 0, \tag{1}$$

is considered, where $u_0 \in X$, X is complex Banach space, $\beta : (0, +\infty) \to (1, 2)$ is the integration order, the linear operator $\partial_t^{-\beta(\cdot)}$ stands fractional integral in time of order $\beta(\cdot)$, and $A : D(A) \subset X \to X$ is an unbounded linear operator of sectorial type. For the sake of the simplicity it is assumed that $f \equiv 0$.

First of all a discussion on the convenience of a definition of the operator $\partial_t^{-\beta(\cdot)}$ versus some others considered in the literature is shown.

Once a convenient definition is chosen, conditions on $\beta(\cdot)$ for the well–posedness of (1) are stated in the framework of sectorial operators [1]. Under these requirements the asymptotic behavior and the regularity of the solution u is studied. Moreover, the asymptotic behavior of a convolution quadrature based time discretization is studied as well.

All these results extend the corresponding results for the constant order case [2], i.e. the case of $\beta(t) \equiv \beta \in (1, 2)$ is constant.

Theoretical results are numerically illustrated by means of several practical experiments.

Keywords: Fractional integrals; variable order; asymptotic behavior.

AMS Classification: 45A05, 45E10, 45N05.

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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CoPO, the Corrector of Periodic Orbits algorithm with high precision

A. Dena

SUMMARY

In this talk the CoPO (cf. [1]) algorithm is presented. This corrector algorithm computes up to any arbitrary precision periodic orbits of dynamical systems. It is based on an optimized shooting method combined with a numerical ODE solver, TIDES (cf. [2]), that uses a Taylor series method. This methodology is nowadays the only one capable to reach precisions up to 1000 digits or more. Moreover, some numerical tests for the Henon-Heiles' Hamiltonian and the Lorenz's model which show the good behavior of the proposed method are presented. Finally, a data base (cf. [3]) of rigorous and high precision periodic orbits of the Lorenz model is exposed.

Keywords: Periodic Orbits, shooting method, Taylor series method, Lorenz model.

AMS Classification: 37M20, 65P20

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Travelling waves in the Fisher-KPP equation with nonlinear diffusion and a non-Lipschitzian reaction term

Pavel Drábek[1], Peter Takáč[2]

SUMMARY

We consider a one-dimensional reaction-diffusion equation of Fisher-Kolmogoroff-Petrovsky-Piscounoff type. We investigate the effect of the interaction between the nonlinear diffusion coefficient and the reaction term on the existence and nonexistence of travelling waves. Our diffusion coefficient is allowed to be degenerate or singular at both equilibrium points, 0 and 1, while the reaction term need not be differentiable. These facts influence the existence and qualitative properties of travelling waves in a substantial way. See [1] for preliminary version of this work.

Keywords: Fisher-Kolmogoroff-Petrovsky-Piscounoff equation; travelling wave; degenerate and/or singular diffusion; non-smooth reaction term; existence and nonexistence of travelling waves; an overdetermined first-order boundary value problem.

AMS Classification: 35Q92, 92D25, 34B08

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Raviart-Thomas finite elements of Petrov-Galerkin type

F. Dubois[1], **I. Greff**[3], **Ch. Pierre**[3]

SUMMARY

The mixed finite element method for the Poisson problem with the Raviart-Thomas elements of low-level can be interpreted as a finite volume method with a non local gradient. In this contribution, we propose a variant of Petrov-Galerkin type for this problem to ensure a local computation of the gradient at the interfaces of the elements. The shape functions are the Raviart-Thomas finite elements. Our goal is to define test functions that are in duality with these shape functions: Precisely, the shape and test functions will be asked to satisfy a L^2-orthogonality property. The general theory of Babuï£¡ska brings necessary and sufficient stability conditions for a Petrov-Galerkin mixed problem to be convergent. We propose specific constraints for the dual test functions in order to ensure stability. With this choice, we prove that the mixed Petrov-Galerkin scheme is identical to the four point finite volumes scheme of Herbin, and to the mass lumping approach developed by Baranger, Maitre and Oudin. Finally, we construct a family of dual test functions that satisfy the stability conditions. Convergence is proven with the usual techniques of mixed finite elements.

Keywords: mixed finite elements, finite volumes

AMS Classification: 65N08, 65N12, 65N30

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Symmetric periodic orbits in a four-body problem

Antonio Elipe

SUMMARY

The planar motion of an infinitesimal particle in a restricted four-body problem where the three primaries are in a collinear central configuration is considered. The evolution of the families of symmetric periodic orbits when the mass parameter μ increases from the value $\mu = 0$ has been studied. When the central body is introduced, some new families appear and others change. Spiral points, where the Asymptotic orbits that connect both triangular equilibrium points end at the so called spiral points. The number and the evolution of these points have been analyzed.

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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On the existence of solutions for a nonlinear stochastic partial differential equation arising as a model of phytoplankton aggregation

Nadjia El Saadi[1], Zakia Benbaziz[2],

SUMMARY

In this paper, we are interested in the analytical study of a nonlinear Stochastic Partial Differential Equation (SPDE) arising as a model of phytoplankton aggregation. This SPDE has been obtained in previous studies (cf. [3] and [4]) as a continuum limit from an individual-based model that takes into account small scale biological mechanisms for phytoplankton cells such as random dispersal of cells modelled by Brownian motions, mutual interactions between phytoplankton cells due to their chemosensory behavior and the demographical proces (cell division and death) modelled by a critical branching process. The obtained SPDE is a diffusion equation with a chemotaxis term and a multiplicative branching noise. We establish the existence of solutions (mild solutions) using weak and tightness arguments.

Keywords: Phytoplankton aggregation, Nonlinear stochastic partial differential equation, chemoraxis, Semigroups, Gaussian space-time white noise, Weak convergence, Tightness, Skorohod representation theorem

AMS Classification: 60H15, 47D03

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Reactive flows at pore scale with hybrid computing

Jean-Matthieu Etancelin[1,2], Philippe Poncet[1,3],

SUMMARY

In this work, we aim at producing a generic simulation code that operates at large scale on hybrid (CPU-GPU) clusters for solving reactive micro-flows at pore scale.

The purpose of this talk is to describe how we exploit a semi-Lagrangian method based on a Remeshed Vortex Method [1]. The key feature of this method is to introduce meshless particles as the domain discretisation. These particles evolve along flow characteristics and carry the vorticity of the flow. The vorticity transport-diffusion equation is combined with a Poisson problem to recover a velocity field from the vorticity. In addition, we use a viscous splitting in order to solve convective terms in a Lagrangian framework and the other terms (stretching, diffusion, Poisson, or chemical reactions) in Eulerian framework on an underlying cartesian grid.

The proof of concept and the usability of these methods has been investigated since several years [1], especially using GPU [2] and more recently using a directional splitting and high order remeshing (particle-grid interpolation) schemes [3] on large hybrid computing resources [4].

In the application context of hydrogen, methane or carbon dioxide injection in specific geological layers for storage purposes, a fine models validation is mandatory. Gases are replacing pore fluids in rocks and are interacting with the medium. These physical and chemical phenomena that can alter the operation of the storage by modifying the chemical composition of fluids and the structures of the rocks. Numerical illustrations of this application will be presented together with the computational performances.

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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On the application of novel 2D techniques to model streamflow generation in response to rainfall

J. Fernández-Pato[1,2], J.L. Gracia[3], P. García-Navarro[1],

SUMMARY

An accurate modeling of the rainfall-infiltration-runoff partitioning is one of the key points in hydraulic/hydrologic simulation, specially in long duration events, where the cumulative errors may be relevant. In particular, the infiltration/exfiltration processes become very important in the water balance computation as they serve as connection between both surface and groundwater flows. Hence, in the last decades a lot of effort has been made to accurately predict this phenomenon. Several models to calculate soil infiltration rates have been proposed in the literature over the past 100 years (Horton, Phillip, Green-Ampt, etc.) but a new hypothesis based on the use of fractional derivatives is challenging the traditional infiltration laws.

In this work, a novel application of the fractional derivatives to the Green-Ampt infiltration method [1, 2, 3] is combined with a 2D overland flow model based on the shallow water equations in order to predict the surface water losses due to this phenomenon. This allows to improve previous published results in the literature for several rainfall events on catchments where the infiltration process occurs. The surface flow model is connected to a 2D depth-averaged groundwater flow and tested in several challenging situations with dry-wet fronts and phreatic level disconnection from the surface.

Keywords: hydrological modeling, numerical methods, fractional derivatives

AMS Classification: 76M12, 76S05, 35Q35

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Asymptotic behaviour of the Swallowtail catastrophe

Chelo Ferreira[1], José Luis López[2], Ester Pérez Sinusía[1]

SUMMARY

We analyze the integral representation of swallowtail catastrophe

$$\Psi(x,y,z) := \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}(t^5+xt^3+yt^2+zt)}dt$$

for large values of one of the variables and bounded values of the other two. The integrand of this integral oscillates wildly and the asymptotic analysis is subtle. The standard saddle point method is complicated, therefore we use the simplified saddle point method introduced in (cf. [1]). The analysis is more straightforward with this method and it is possible to derive complete asymptotic expansions of $\Psi(x,y,z)$ for large values of the asymptotic variable and fixed values of the rest. The asymptotic analysis requires the study of different regions separated by the corresponding Stokes lines. The accuracy and the asymptotic character of the approximations is illustrated with some numerical experiments.

Keywords: Swallowtail integral, Asymptotic expansions, Modified saddle point method.

AMS Classification: 33E20, 41A60

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Two-point Taylor expansions in singular one-dimensional boundary value problems: Application to the spheroidal wave equation

Chelo Ferreira[1], José L. López[2], Ester Pérez Sinusía[1]

SUMMARY

We consider the second-order linear differential equation $(x^2-1)y''+f(x)y'+g(x)y=h(x)$ in the interval $(-1,1)$ with initial conditions or boundary conditions (Dirichlet, Neumann or mixed Dirichlet-Neumann). The functions $f(x)$, $g(x)$ and $h(x)$ are analytic in a Cassini disk $\mathcal{D}_r$ with foci at $x=\pm 1$ containing the interval $[-1,1]$. Then, the two end points of the interval may be regular singular points of the differential equation. The two-point Taylor expansion of the solution $y(x)$ at the end points ± 1 is used to study the space of analytic solutions in $\mathcal{D}_r$ of the differential equation, and to give a criterion for the existence and uniqueness of analytic solutions of the boundary value problem. The method is constructive and provides the two-point Taylor approximation of the analytic solutions when they exist. We apply it in the study of eigenfunctions of the spheroidal wave equation.

Keywords: second-order linear differential equations, regular singular point, Frobenius method, two-point Taylor expansions, spheroidal wave equation.

AMS Classification: 34A25, 34B05, 41A58

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Slopes of colored links

Vincent Florens[1]

SUMMARY

We define the slope of a colored link associated to admissible characters on the link group. Away from a certain singular locus, the slope is a rational function which can be regarded as a multivariate generalization of the Kojima–Yamasaki η-function. It is the ratio of two Conway potentials, provided that the latter makes sense; otherwise, it is a new invariant. We present several examples and discuss the invariance by concordance. (joint with A. Degtyarev and A.G. Lecuona)

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Canonical Constants in a Problem of Radzievskij

Luis Floría

SUMMARY

V. V. Radzievskij [4] investigated the motion of two point masses inside a homogeneous spherical cloud, assuming that the mutual interactions between these masses and between them and the particles of the cloud are described by their Newtonian gravitational attractions. In addition to this, the constant density of the cloud is supposed to be sufficiently small so that the resistance of this material medium to the motion can be neglected.

The problem of relative motion of the said point masses can be brought into the form of a perturbed Keplerian system, in which the perturbing force is a conservative central force. With the help of the first integrals of the angular momentum and the energy, this author followed the conventional solution procedure in plane polar coordinates, formally leading to an orbit equation (in inverted form), $\varphi = \varphi(r\,;r_0\,,\varphi_0)$, in terms of *an Abelian integral.*

D. Mihailović tackled different analytical treatments of Radzievskij's problem: reading it as a perturbed Kepler problem ([1], [2]) he used vector elements and the corresponding perturbation equations, in line with the developments of the Milanković School of Analytical and Celestial Mechanics; regarding the problem as a forced harmonic oscillator ([3]), he resorted to the method of variation of arbitrary constants to study a particular solution to the second–order non–homogeneous linear differentiall equation governing that oscillator.

In this talk *we introduce canonical constants and orbital elements for this problem.*

Keywords: Radzievskij's three–body problem, perturbed Keplerian systems, conservative central force, first integrals, canonical constants of motion.

AMS Classification: 70 F 05, 70 H 15, 70 H 20

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Filling holes using a mesh of filled curves

M.A. Fortes[1], **P. González**[1], **A. Palomares**[1], **M.L. Rodríguez**[1]

SUMMARY

The objective of this talk is to fill graphics of surfaces with holes-meeting shape conditions, i. e., we want to determine values of a surface inside a hole -where it is not defined- by means of its values outside the hole -where it is properly defined- in such a way that the final reconstructed surface be fair and smooth enough. The procedure considered to get this aim is based on a one-dimensional hole-filling problem, leading to a kind of "wireframe" surface. We develop the theoretical aspects of the problem and we show some graphical examples to illustrate the proposed method.

Keywords: Filling holes, minimal energy, Powell-Sabin

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Error Analysis of non inf-sup Stable Discretizations of the time-dependent Navier-Stokes Equations with Local Projection Stabilization

Javier de Frutos, Bosco García-Archilla[1], Volker John[2], Julia Novo[3]

SUMMARY

In this talk we consider non inf-sup stable finite element approximations to the evolutionary Navier–Stokes equations. Several local projection stabilization (LPS) methods corresponding to different stabilization terms are analyzed, thereby separately studying the effects of the different stabilization terms. Error estimates are derived in which the constants are independent of inverse powers of the viscosity. For one of the methods, using velocity and pressure finite elements of degree l, it will be proved that the velocity error in $L^\infty(0,T;L^2(\Omega))$ decays with rate $l+1/2$ in the case that $\nu \leq h$, with ν being the dimensionless viscosity and h the mesh width. In the analysis of another method, it was observed that the convective term can be bounded in an optimal way with the LPS stabilization of the pressure gradient. Fully discrete schemes with both the implicit Euler and the Crank-Nicolson methods are considered and analyzed Some numerical studies confirm the analytical results.

Keywords: Navier-Stokes equations, LPS stabilization

AMS Classification: 65M12, 65M60

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Convergence and error estimates for the compressible Navier-Stokes equations

T. Gallouët

SUMMARY

In this talk we will consider some numerical schemes for the computation of approximate solutions of the compressible Navier-Stokes equations with a perfect gas equation of state. In the evolution case, we will give some error estimates when the solution of the continuous system is regular enough. Without regularity hypothesis on the solution of the continuous system, we will present some convergence results (up to subsequence) of the approximate solutions to an exact solution, essentially for the stationary case.

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Stabilized mixed methods for convection-diffusion problems

María González[1], Magdalena Strugaru[2]

SUMMARY

We are interested in the numerical approximation of the solution to the scalar convection-diffusion equation in mixed form. This problem was previously analyzed by Douglas and Roberts [1], Jaffre [4] and Thomas [5]. One of the difficulties that arise is the need that the discrete subspaces satisfy the discrete inf-sup condition. We will present stabilized dual-mixed finite element methods that allow to avoid this requirement. We will explore the existence and uniqueness of a solution to the stabilized mixed formulation, and the derivation of a priori and a posteriori error estimates. Numerical experiments illustrating the performance of the method will be shown.

Keywords: convection-diffusion, mixed finite element, stabilization, a posteriori error estimates

AMS Classification: 65N30, 65N12, 65N15

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Development of a control tool for releases of pollutants in rivers

Geovanny Gordillo[1], Mario Morales-Hernández[1,2], Pilar García-Navarro[1]

SUMMARY

The objective of this work is to control the environmental impact produced by point and non-point sources discharged to natural bodies of water. The optimal control problem for a pollutant in 1D is based on a gradient-based method. This technique involves to solve the convection-diffusion equation of the transport of a pollutant in rivers and streams [1]

$$\frac{\partial \phi}{\partial t} + \frac{\partial (u\phi)}{\partial x} - E\frac{\partial}{\partial x}\left(\frac{\partial (u\phi)}{\partial x}\right) = f(x,t) \tag{1}$$

where f is a a controllable source to adjust the concentration (ϕ) to a given objective function defined at a target location J, and u the flow velocity. And its adjoint equation:

$$-\frac{\partial \sigma}{\partial t} - \frac{\partial (u\sigma)}{\partial x} - E\frac{\partial}{\partial x}\left(\frac{\partial (u\sigma)}{\partial x}\right) = (\phi - \phi_{obj}) \tag{2}$$

This system of equations is solved by means of an explicit first order upwind scheme, considering backward temporal-integration for the adjoint equation to minimize the error. The gradient descent method can be formulated as:

$$f^{k+1} = f^k + \varepsilon \nabla J^k \tag{3}$$

In order obtain global convergence, not only the step lengths must be chosen properly, but also the search direction. Such that $J^{k+1} < J^k$. The sensitivity of J with respect to the perturbations in the source terms f can be expressed as:

$$\nabla J(x,t) = \frac{\delta J}{\delta f(x,t)} = -\sigma(x,t) \tag{4}$$

The term σ is the sensitivy vector, which is used to adjust the concentration at any other point.

Keywords: Pollutant, optimization, gradient method

AMS Classification: 35L02, 35L03, 35Q35

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Elliptic problems involving a gradient term with natural growth

Jean Pierre Gossez

SUMMARY

We consider the problem

$$-\Delta u = g(u)|\nabla u|^2 + f(x,u) \text{ in } \Omega, u > 0. \text{ in } \Omega, u = 0. \text{ on } \partial\Omega,$$

where Ω is a smooth bounded domain in $\mathbb{R}^{\mathbb{N}}$. Using a Kazdan-Kramer change of variable, this problem can be reduced to a semilinear problem without gradient term, which can then be approachable by variational methods. However various new difficulties arise, in particular with respect to the Ambrosetti-Rabinowitz condition. We investigate some of these difficulties.

In the second part of the talk, we consider the situation of a system. Using again a Kazdan-Kramer change of variables, this system can be transformed into a system without gradient term. In some cases, this latter system can be handled by an upper-lower solution approach, by variational methods, or by blow-up. The blow-up procedure leads to the study of new Liouville type theorems.

Joint work with D. de Figueiredo (Campinas, Brasil), H. Ramos Quoirin (Cordoba, Argentina) and P. Ubilla (Santiago, Chile).

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Multigrid Waveform Relaxation Based On Finite Element Discretisation

Noora Habibi

SUMMARY

We propose the waveform relaxation method and its multigrid acceleration as solution of ordinary differential equations based on linear basis Finite Element discretisation and by regular triangular mesh ($\beta = \frac{\pi}{2}$ introduced in [1]). Its convergence analysis by means of semialgebraic mode analysis (SAMA) are investigated. Furthermore, quantitative convergence estimates and several numerical results in two-dimension are presented.

Keywords: Finite Element method, Waveform relaxation method, Multigrid technique, Semialgebraic mode analysis.

AMS Classification: First code, second, third

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Staggered schemes for compressible flows

Raphaèle Herbin

SUMMARY

In recent years, a class of schemes has been developed for the numerical resolution of fluid flow in the context of nuclear safety. These flows are complex in nature in that they may include high and low Mach zones. Staggered schemes involve discrete velocity unknowns at the faces and scalar unknowns at the center of the cells and are naturally stable for incompressible regimes; they are thus well qualified for the discretisation of such flows. The well known Marker And Cell (MAC) scheme is an example of such a staggered known Marker And Cell (MAC) scheme is an example of such a staggered scheme for rectangular meshes. The presentation will deal with this class of schemes for the compressible Euler equations. The main features of the schemes, besides the use of a staggered mesh, is the discretisation of the (non-conservative) internal energy balance rather than the total energy, and an unwinding of the fluxes relative to the material speed, therefore natively ensuring the positivity of the density and the internal energy. Time discretization can be explicit or performed by a pressure correction technique. In both cases, a correction term is added to the discrete internal energy equation which allows to recover the correct shock speeds. Moreover, the schemes are proven to be consistent in the Lax sense: indeed, provided compactness assumptions, the sequence of approximate solutions can be shown to converge to a weak solution of the Euler equations. A discrete entropy inequality is obtained when the discretization of the mass balance and internal energy equations is implicit and the fluxes upwind. For explicit discretisations, the current theoretical results require a constraint on the time step that is stronger than the CFL constraint or a specific stabilization term. It is worthwhile noticing that for a constant density, the semi- implicit scheme degenerates to a standard algorithm for the incompressible regime. In fact, in the case of the compressible barotropic Navier-Stokes equations, it has been shown that for a fixed mesh, the numerical solution of the compressible scheme converges to the numerical solution of the incompressible scheme as the Mach number tends to zero.

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Flat and Compact Support Solutions to some Semilinear Elliptic Problems with Non-Lipschitz Nonlinearities

Jesús Hernández

SUMMARY

We give a general overview of some recent results concerning existence and properties of non-negative solutions to some semilinear elliptic boundary value problems. These problems may exhibit smooth positive solutions but also positive solutions with zero normal derivative(?flat? solutions) and compact support solutions. We study in particular these flat solutions with emphasis on its stability properties. It turns out that in some cases the stability depends on the space dimension.

This is joint work with J.I. Díaz and Y. Ilyasov.

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Vortex-based penalized method for permeability estimation of real samples

Laurène Hume[1,2], Philippe Poncet[1,3],

SUMMARY

The investigation of fluid flow at the pore scale, based on X-ray scans of real samples, is one of the most challenging problems in CFD. Flow simulation at this scale is difficult because of the inherent complexity of the geometry, which includes a fluid-solid interface with possible roughness. The high resolution needed to capture relevant geometrical details has to be handled without using tremendous memory resources, excluding traditional assembling methods like finite elements or finite volumes. In our study, we use a robust hybrid grid-particle method [1, 2] to solve the advection-diffusion problem. The complete problem writes in the domain Ω

$$\begin{cases} u = \bar{u} \text{ in solid domain,} \\ -\text{div}(2\mu(\alpha, u)D(u)) = f - \nabla P \text{ in fluid domain ,} \\ \text{div} u = 0 \text{ in } \Omega, \\ \partial_t \alpha + u \cdot \nabla \alpha - \eta \Delta \alpha = 0 \text{ in } \Omega, \end{cases} \tag{1}$$

where u is the fluid velocity, $D(u) = (\nabla u + \nabla u^T)/2$, μ is the fluid viscosity, f is the external force, P is the pressure, α is a transported quantity of interest and η is its diffusion coefficient. In [3], we use the penalized version of (1) to get existence and unicity of the solution. The numerical method that allows permeability prediction [4] is then built with a velocity/vorticity splitting for Stokes solution.

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On the uniqueness of minimisers of Ginzburg-Landau functionals

Radu Ignat

SUMMARY

We provide necessary and sufficient conditions for the uniqueness of minimizers of the Ginzburg-Landau functional for vector valued maps with a boundary data that is non-negative in a fixed direction. Furthermore, we show that, when minimizers are not unique, the set of minimizers is generated from any of its elements using appropriate orthogonal transformations. We also prove corresponding results for harmonic maps.

This is a joint work with L. Nguyen (Oxford), V. Slastikov (Bristol) and A. Zarnescu (Bilbao).

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Stabilized virtual element method for the incompressible Navier-Stokes equations

Diego Irisarri[1], Guillermo Hauke[1]

SUMMARY

In this work, we present a stabilized virtual element method (VEM) discretization for the incompressible Navier-Stokes equations. Basically, VEM can be considered a generalization of FEM that enables a polynomial decomposition of the domain. VEM has been applied to elasticity and fluidmechanics differential equations [1, 2, 3]. In this work, the concepts of stabilized methods [4] are introduced in the VEM formulation. Thus, stabilization terms are included in the variational form to circumvent the Babuška-Brezzi condition and to stabilize the solution for convection dominated flows. Numerical examples are presented to show the behavior of the method.

Keywords: VEM, Navier-Stokes equations, Stabilized methods

AMS Classification: 76D05, 65M60

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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On a bounded distribution derived from the shifted Gompertz law

P. Jodrá

SUMMARY

A two-parameter distribution with bounded domain is obtained by means of an exponential transformation of the shifted Gompertz model (cf. [1]). The new distribution has probability density function given by

$$f(x;\alpha,\beta) = \beta\left[1+\alpha(1-x^{\beta})\right]x^{\beta-1}\exp\left(-\alpha x^{\beta}\right), \qquad 0<x<1, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are two shape parameters.

The main statistical properties of the distribution under consideration are provided in closed form. The moments are written in terms of the incomplete gamma function and the quantile function in terms of the Lambert W function. The order statistics are also studied and, in particular, the new model belongs to the do main of maximal/minimal attraction of the Weibull law. Moreover, the members of the new family of distributions can be ordered in terms of the hazard rate order.

The parameter estimation is carried out by the methods of maximum likelihood, least squares, weighted least squares and quantile least squares. The performa nce of these methods is assessed by means of a Monte Carlo simulation study. A symptotic confidence intervals based on maximum likelihood together with par ametric bootstrap confidence intervals are discussed and the coverage probabilit y and average width of these intervals are studied.

The random variable defined by (1) is useful to model real data taking values in the standard unit interval, such as percentages and p roportions, and its practical usefulness is illustrated by means of several real data sets.

The proposed distribution may provide a better fit than other two-parameter distributions, such as the classical beta and Kumaraswamy distributions as well as other models recently introduced (cf. [2] and [3]).

Keywords: Beta, Kumaraswamy, shifted Gompertz, bounded support.

AMS Classification: 60E05, 62P10, 33B30.

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Bifurcations in flight dynamics and aeroelasticity: Nonlinear analysis and numerical simulations

Sébastien Kolb[1]

SUMMARY

In aeronautics some phenomena require a nonlinear approach because the linear analysis is not suffient to catch the underlying physics. Some issues met in the fields of flight dynamics and aeroelasticity are concerned with this feature. This study aims at showing so-called bifurcations implying unpredictable behaviours in the linear frame such as jumps or appearances of limit cycles and thus for which a nonlinear analysis is mandatory in order to catch the real behaviour. The methodology and practical aspects necessary to perform such an analysis are here exposed.

The first topic concerns the longitudinal flight of a F-18 fighter. The results of the computation with the numerical bifurcation analysis toolbox *matcont* are shown and the angles-of-attack α (equilibria and limit cycles) are given in function of the elevator deflection δ_e with a fixed throttle. A Hopf bifurcation is diagnosed and gives raise to periodical orbits which may surprise the pilot. This can be a hazardous situation to manage (especially with a nonzero flight-path angle during a landing phase). Moreover there is a range of elevator deflections δ_e for which there are multiple equilibria for the same elevator deflection. A pitchfork bifurcation is responsible for a hazardous stabilization at a nonzero bank angle.

The other topic deals with the aeroelasticity of an airfoil. The nonlinear physics come from the pitch stiffness (torsion), the plunge stiffness (flexion) or the aerodynamics (stall) amongst others. Several approaches are used so as to assess the flutter properties. On the one side, a continuation algorithm (*matcont* toolbox of *matlab*) allows the nonlinear analysis by computing the equilibria and the envelope of the periodical orbits. On the other side, a software of numerical simulation (*star-ccm+*) is exploited with a method of *overset mesh* so as to take into account easily the motions of plunge and pitch of the airfoil and may help investigating some types of nonlinear behaviour.

For example, the plunge stiffness can be hardened $k_h(h) = K_h\left(1 + \xi h^2\right)$. The Hopf bifurcation associated to $\xi = 50$ is supercritical and the one associated to $\xi = 0.09$ is subcritical (more dangerous) as observed in the bifurcation diagrams. Thus the hardening of the plunge stiffness seems to have a favourable effect.

To put in a nutshell several ways to analyse these nonlinear phenomena met in aeronautics are shown, diverse mathematical tools are used and reveals to be helpful for the design of an airplane.

Keywords: bifurcation theory, flight dynamics, aeroelasticity, overset mesh

AMS Classification: 34K18, 37G10, 37G15, 65N08

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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A multigrid multilevel Monte Carlo method for transport in the Darcy-Stokes system

Prashant Kumar

SUMMARY

In this talk, a multilevel Monte Carlo (MLMC) method for Uncertainty Quantification(UQ) of advection-dominated contaminant transport in a coupled Darcy-Stokes flowsystem will be presented. In particular, we will focus on high-dimensional epistemicuncertainty due to an unknown permeability field in the Darcy domain that is modeledas a lognormal random field. Different numerical strategies are explored for the subproblemsand an optimal combination for the MLMC estimator is suggested. Specifically,we propose a monolithic multigrid algorithm to efficiently solve the steady-stateDarcy-Stokes flow with a highly heterogeneous diffusion coefficient. Furthermore,we describe an Alternating Direction Implicit (ADI) based time-stepping for the fluxlimitedquadratic upwinding discretization for the transport problem. Numerical experimentsillustrating the performance of the multigrid MLMC method with respect tothe smoothness of permeability field will be discussed in detail.

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Periodic Solutions in the Hénon-Heiles Rotating System

V. Lanchares, M. Iñarrea, A.I. Pascual, J.P. Salas[1], J. Palacián, P. Yanguas[2]

SUMMARY

Hénon–Heiles system is probably one of the most studied dynamical systems, because it can be used to model different physical problems and also to highlight different properties inherent to most of two degrees of freedom nonlinear Hamiltonian systems. It arose as a simple model to find additional conservation laws in galactic potentials with axial symmetry [1]. In the context of galactic dynamics, to study stellar orbits, the rotation of the galaxy must be taken into account [2] so that it makes sense to consider a generalized Hénon-Heiles system in a rotating frame. Our aim is to prove the existence of periodic orbits in a neighborhood of the origin for appropriate values of the rotating frequency. To this end, we use normal form theory to demonstrate that the number of periodic orbits is in correspondence with the equilibrium solutions of the original system, with the same type of stability.

Keywords: Generalized Hénon-Heiles system, periodic orbits, normal forms

AMS Classification: 70H08, 70H09, 70H12, 70H15, 34C25, 37C27

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Uniformly convergent expansions of the Struve functions in terms of elementary functions

J. L. López[1], **P. J. Pagola**[1], **P. Palacios**[2]

SUMMARY

Struve functions are solutions to the non-homogeneous Bessel's differential equation

$$\frac{d^2w}{dz^2}+\frac{1}{z}\frac{dw}{dz}+\big(1-\frac{\nu^2}{z^2}\big)w=\frac{(z/2)^{\nu-1}}{\sqrt{\pi}\Gamma(\nu+1/2)}.$$

They are useful in the description of several phenomena in aerodynamics, quantum mechanics, optical diffraction and other physical areas.

The approximation of these functions in terms of elementary functions is quite convenient in the analysis of those physical phenomena. The most commonly used approximations are the Taylor and the asymptotic expansions, useful for small and large values of $|z|$ respectively. But these approximations are not uniformly valid for $\Re z \in \mathbb{R}$. Then, we derive a convergent series expansion of the Struve functions in terms of elementary functions of z that, conveniently scaled, hold uniformly in $\Re z \in \mathbb{R}$.

The starting point of the analysis is a convenient integral representation. Then, the Taylor expansion of an appropriate factor of the integrand is used. The uniform expansions derived are accompanied by realistic error bounds and are compared with the Taylor and asymptotic expansions.

Keywords: Struve functions; convergent expansions; uniform expansions.

AMS Classification: 33C47; 41A58

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Shape preserving properties of general class of bases and accurate computation

E. Mainar[1], **J.M Peña**[2], **B. Rubio**[3]

SUMMARY

We present a very general procedure for generating, from an initial system and a positive function φ, new systems of functions useful for curve design. These systems, which we call weighted φ-transformed systems, include important rational bases (see [1], [2]) as well as systems belonging to spaces mixing algebraic, trigonometric and hyperbolic polynomials, which are useful in many applications, for instance in Isogeometric Analysis (cf. [3]). The weighted φ-transformed systems inherit from the initial system its nice geometric properties and its accuracy when computing with its collocation matrices.

We show many general classes of spaces that can be generated by weighted φ-transformed systems. In particular, our results allow us to deduce shape preserving properties of general rational functions that cover the family of bases introduced in [1]. We illustrate a Casteljau type algorithm for evaluating by means of corner cutting algorithms and we also illustrate numerical examples showing the accurate solution of linear systems associated to the collocation matrices of weighted φ-transformed systems.

Keywords: Normalized Totally Positive Basis, Normalized B-basis, Rational basis, Bidiagonal decompositions, Accurate computations.

AMS Classification: 65D17, 65F05, 65D05, 41A05, 42A10

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Laplacian (co)flow of a locally conformal parallel G_2-structure

V. Manero[1], A. Otal[2], R. Villacampa[2]

SUMMARY

The development of flows in Riemannian geometry has been mainly motivated by the study of the Ricci flow. However, the same techniques are also useful in the study of flows involving other geometrical structures, like for example, the Kähler Ricci flow. Concerning flows on G_2-manifolds, for any closed G_2-structure σ_0 on a manifold M, Bryant (Proceedings of Gökova Geometry-Topology Conference 2005, 2006) introduced a natural flow, the so-called *Laplacian flow*, given by

$$\begin{cases} \frac{d}{dt}\sigma(t) = \Delta_t\sigma(t), \\ \sigma(0) = \sigma_0, \quad d\sigma(t) = 0, \end{cases}$$

where Δ_t is the Hodge Laplacian operator of the metric determined by $\sigma(t)$. The short time existence and uniqueness of solution for the Laplacian flow of any closed G_2-structure, on a compact manifold M, has been proved by Bryant and Xu in the unpublished paper arxiv:1101.2004[math.DG].

Karigiannis, McKay and Tsui (Diff. Geom. Appl. 2012) introduced the *Laplacian coflow*. In this case the initial G_2-form is claimed to be coclosed, i.e. $d*\sigma_0 = 0$. Up to now, short time existence of solution of the coflow is not known. Assuming short time existence and uniqueness of solution, the authors show that the condition of the initial G_2-form σ_0 to be coclosed (equiv. ψ_0 closed) is preserved along the flow.

Here we are concerned with studying the Laplacian flow, resp. coflow, of an LCP G_2-structure on a manifold M defined as:

$$\begin{cases} \frac{d}{dt}\sigma(t) = \Delta_t\sigma(t), \\ \sigma(0) = \sigma_0, \\ d\sigma(t) = 3\,\tau(t) \wedge \sigma(t), \\ d*_t\,\sigma(t) = 4\,\tau(t) \wedge *_t\sigma(t). \end{cases} \qquad \begin{cases} \frac{d}{dt}\psi(t) = -\Delta_t\psi(t), \\ \psi(0) = \psi_0, \\ d\psi(t) = 4\,\tau(t) \wedge \psi(t), \\ d*_t\,\psi(t) = 3\,\tau(t) \wedge *_t\psi(t). \end{cases}$$

The first examples of long time solutions of these flows are given. Our examples are one-parameter families of Locally Conformal Parallel G_2-structures on solvable Lie groups. We start finding solutions for the Laplacian flow and the found solutions are used to construct long time solutions to the Laplacian coflow starting from a Locally Conformal Parallel structure. These results can be found in the preprint available in arxiv:1711.08644[math.DG].

Keywords: Geometric flows, G_2-structures, locally conformal parallel structures,...

AMS Classification: 53C38, 53C25, 22E25

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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On some minimization problems in $\mathbb{R}^N$

Mihai MARIŞ

SUMMARY

We present some recent improvements of the concentration-compactness principle and show that they give a new insight in some minimization problems arising in the study of solitary waves for nonlinear dispersive equations. We consider both local and nonlocal equations having a Hamiltonian structure.

Keywords: constrained minimization, ground states, concentration-compactness principle

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Equilibrium and non-equilibrium models applied to unsteady sediment transport

S. Martínez-Aranda[1,*], J. Murillo[1], P. García-Navarro[1]

SUMMARY

Sediment transport is ubiquitous in river and coastal morpho-dynamical systems and its influence on the global behaviour of these environmental systems can be decisive. Usually, sediment transport is divided into suspended and bed load, representing the different physical processes involved in the sediment movement. When only bed load is considered, the morpho-dynamical system can be mathematically modelled by the well-known shallow water equations for the hydrodynamic component coupled with a continuity equation for the solid mass, called Exner equation [1]. Exner equation is based on equilibrium hypothesis, which considers that the actual bed load rate is equal to the transport capacity of the flow, determined by one of the experimental closure relations reported in literature. This hypothesis is fairly admitted in steady and quasi-steady flow conditions, however for highly unsteady events, as floods or dyke failures, the actual bed load transport rate suffers a temporal and spatial lag respect to the flow capacity and the equilibrium formulation is no longer valid [2]. The non-equilibrium approaches have received increasing attention in the last years and have demonstrated to be suitable for modelling both highly erosive and scour phenomena.

In this work, the equilibrium and non-equilibrium formulations are applied to erosive problems involving unsteady flows and the presence of non-erodible layers. The system of equations is solved coupled by means of a Finite Volume Method (FVM) based on a Roe's approximated upwind scheme [3, 4]. The numerical model is robust and conservative. The equilibrium and non-equilibrium formulations are compared in terms of numerical prediction accuracy for these kind of complex erosive problems.

Keywords: Hyperbolic conservation laws systems, finite volume methods, non-equilibrium sediment transport, non-erodible layers.

AMS Classification: 35-Q35, 76-M12, 86-A05

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Generalized B-Splines: Structure and Properties

Peter Massopust

SUMMARY

The classical polynomial and exponential cardinal B-splines are generalized to include complex and Clifford-valued orders. We highlight some properties of these new B-splines and exhibit connections to fractional integrals and derivatives. The problem of cardinal interpolation is addressed in the case of Clifford-valued polynomial B-splines and a sampling and interpolation theorem is derived.

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About A Time Discretization of A New Mathematical Model of Two-Phase Flow in Nanoporous Media

Imane MELZI[1], Youcef ATIK[2]

We propose a mathematical model for two-phase flow in nanoporous media. Unlike classical models, our model suppose the relative permeabilities depend not only on saturation but also on pressure or the gradient of pressure. Using usual laws of flows in porous media we obtain a system of two nonlinear partial differential equations, the first is elliptic and the second is parabolic degenerate. To solve our system, we begin by regularizing the degenerate equation by adding a vanishing term to the coefficient causing the degeneracy. The aim of this communication is to prove the existence of a weak solution of a time discrete version of the regularized model.

Keywords: Nanoporous media, Time discretization, Galerkin's approximation

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Oanishing of Semiconformal curvature tensor in spacetime settings

Ali Musavvir[1], Naeem Ahmad Pundeer[2]

SUMMARY

The aim of the present paper is to study the semiconformal curvature tensor for spacetime of general relativity. We establish the subsistence of Killing and con- formally Killing vectors satisfying EFE for the case when semiconformal curvature tensor vanishes. We extend the same case for the study of cosmological models with dust and perfect fluid.

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A Stochastic Square of the Rayleigh Diffusion Process

Ahmed Nafidi, Ghizlane Moutabir[1], Ramón Gutiérrez-Sánchez[2]

SUMMARY

This work describes a study of a new one-dimensional homogeneous stochastic process termed the square of the Rayleigh process. The model is based on the homogeneous stochastic Rayleigh diffusion process (cf. [1]) which is examined from the perspective of a nonlinear stochastic differential equation and used in various aspects of stochastic modelling such as physics, stochastic finance, demographic and economic (cf. [2]).
In this study, we first obtain the transition probability density function of the model after which we determine the trend functions (conditional and non-conditional). Then, the drift parameters are estimated by maximum likelihood on the basis of continuous sampling of the process (cf. [3]). Finally, in the diffusion coefficient, we consider the problem of parameter estimation, doing so by a numerical approximation.

Keywords: Rayleigh diffusion process, Stochastic differential equation, Trend function, Statistical inference in diffusion process.

AMS Classification: 60J60, 60J35, 62M86.

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Statistical inference in a new extension of stochastic Gompertz process with exogenous factors

Ahmed Nafidi, Ghizlane Moutabir[1], R. Gutiérrez Sánchez[2]

SUMMARY

We define a new non-homogeneous extension of the well know Gompertz diffusion process (cf. [1]). This extension is obtained as a combination of the two non-homogeneous versions of the same process: the first was studied by Gutierrez et al. in (cf. [2]), who considered the exogenous factors in the intrinsic growth rate, while the second was defined in (cf. [3]), where the exogenous factors were assumed to affect the deceleration factor of the process. For the new process defined in this paper, the probabilistic characteristics are determined and the statistical inference is studied.

Keywords: Stochastic Gompertz process, Exogenous factors, Statistical inference in diffusion process.

AMS Classification: 60J60, 62M86

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Fractal approximation for 3D periodic data

M.A. Navascués[1], S. Jha[2], M.V. Sebastián[3], A.K.B. Chand[4]

SUMMARY

We study a trigonometric type fitting map for functions or data defined on a grid on the two-dimensional interval $[-\pi,\pi]\times[-\pi,\pi]$, assuming periodicity in both variables. Let us consider a set of three-dimensional data on a grid, for $f\in\mathcal{C}(T^1\times T^1)$ ([1], [2], [3]):

$$\{(x_i,y_j,f(x_i,y_j)) : i=1,2,\dots,m; j=1,2,\dots,n\},$$

where $x_{i+1}-x_i=\pi/m$, $i=1,2,\dots,2m-1$; $y_{j+1}-y_j=\pi/n$; $j=1,2,\dots,2n-1$, and

$$\mathcal{J}_{mn\gamma}(f)(x,y)=K_{mn\gamma}(x,y)\sum_{i=1}^{2m}\sum_{j=1}^{2n}f(x_i,y_j)\left|\frac{\sin\left(\frac{1}{2}m(x_i-x)\right)}{m\sin\left(\frac{1}{2}(x_i-x)\right)}\right|^{\gamma}\left|\frac{\sin\left(\frac{1}{2}n(y_j-y)\right)}{n\sin\left(\frac{1}{2}(y_j-y)\right)}\right|^{\gamma}, \tag{1}$$

where

$$K^{-1}_{mn\gamma}(x,y)=\sum_{i=1}^{2m}\sum_{j=1}^{2n}\left|\frac{\sin\left(\frac{1}{2}m(x_i-x)\right)}{m\sin\left(\frac{1}{2}(x_i-x)\right)}\right|^{\gamma}\left|\frac{\sin\left(\frac{1}{2}n(y_j-y)\right)}{n\sin\left(\frac{1}{2}(y_j-y)\right)}\right|^{\gamma}. \tag{2}$$

These approximants are extended to systems of fractal functions which are smooth or non-smooth depending on the choice of scaling factors. We obtain bounds of the approximation error and the convergence of the fitting functions with very weak conditions, when the sampling frequency is increased. The density of these mappings in the space of two-dimensional periodic and continuous functions is proved.

Keywords: Fractal Interpolation Functions, Two-dimensional Approximation, Fractals

AMS Classification: 28A80, 42A10, 42A15

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Rate of numerical difusion of finite volume schemes

Pascal OMNES

SUMMARY

Dissipation rates of Godunov upwind type schemes are often studied either through Fourier analysis orthrough the so-called modifised equation technique. However, these techniques are limited to uniformcartesian meshes. In this work, we introduce a method based on energy estimates that allows to deriveexplicit lower bounds for the dissipation rate of semi-discrete Godunov type schemes based on nonuniform cartesian, or on triangular meshes. We illustrate our purpose by applying the theory to theone-dimensional transport equation discretized by the upwind scheme on non-uniform meshes and to thetwo-dimensional wave equation discretized by the Godunov scheme on triangular meshes. On rectangularmeshes, we also study a variant of the Godunov scheme which preserves a discrete variant of the waveinvariant space. Several steps are used to obtain these results. First, the discrete solutions of the schemesare decomposed as sums of two elements: the first in the kernel of the discrete transport and waveoperators and the other in the orthogonal of these kernels. Then it is verified whether these orthogonalsubspaces are stable by the discrete operators. Finally, discrete Poincaré inequalities in the orthogonal subspaces are used to obtain the dissipation lower bounds. The best provable lower bounds are in additionshown to be solutions of an eigenvalue problem.

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Numerical analysis and thin layers

Eugene O'Riordan

SUMMARY

Boundary value problems involving singularly perturbed partial differential equations arise in several branches of applied mathematics. The analytical solutions to these problems typically exhibit steep gradients in narrow regions of the domain. The presence of these gradients can have an adverse effect on the performance of classical numerical methods. Some of these shortcomings will be highlighted to demonstrate that alternative perspectives are required when dealing with multi-scale problems. Some of the approaches to designing layer-adapted numerical methods will be introduced and their potential advantages (over a classical approach) will be discussed.

Keywords: Singularlly perturbed, numerical analysis, boundary and interior layers

AMS Classification: 65L12, 65N12, 65N15, 65N06

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Non-associative Algebraic Hyperstructures and its Applications to Biological Inheritance

Oyeyemi O. Oyebola[1], Temitope G. Jayeola[2]

ABSTRACT

In this paper, we investigate the nonassociative properties of algebraic hyperstructures as it plays out in the biological inheritance which is expressed in the genotypic and phenotypic information that are passed to the progeny from the parental traits. The largest class of hyperstructures called H_v-structures are described for the filial generations.

Keywords: H_v-structures, Filial generations...

AMS Classification: 17D92, 92B99, 92D10

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Uniformly convergent expansions of the Generalized Hypergeometric function ${}_pF_q$ in terms of elementary functions

Pedro J. Pagola[1], José Luis López García[1],

SUMMARY

The solution of the following linear equation of order $q+1$

$$\left[z\frac{d}{dz}\prod_{j=1}^{q}\left(\frac{d}{dz}+b_j-1\right) - z\prod_{i=1}^{p}\left(z\,\frac{d}{dz}+a_i\right)\right] w = 0,$$

is the generalized hypergeometric function ${}_pF_q(a_1,...,a_p\,,\,b_1,...,b_q\,;\,z)$ with $p+q$ parameters. This function is useful in statistic and also appear in the evaluation of the so-called Watson integrals which characterize the simplest possible lattice walks. It's also potentially useful for the solution of more complicated restricted lattice walk problems.

In general, the generalized hypergeometric function ${}_pF_q$ has not an explicit expresion and to compute it, we have to use approximation techniques. To this end, the most used are the Taylor or asymptotic expansion, valid for small and large values of the argument z respectively.

In this work, we derive new expansions of the generalized hypergeometric function ${}_pF_q$ in terms of elementary functions of z that converge in different regions, bounded or unbounded, of the complex plane. We give either, explicit formulas for the coefficients of the expansions. Using a integral representation of of ${}_pF_q$, the key point of the analysis is the approximation of an appropriate factor of the integrand.

Finally, we show the accuracy of the approximations by means of several numerical experiments.

Keywords: generalized hypergeometric function, convergent expansions

AMS Classification: 33C20, 41A58, 41A80

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Filling holes of Generalized Offset Surfaces by Biquadratic Splines

M. Pasadas

SUMMARY

Standard Offset surfaces are defined as locus of the points which are at constant distance along the unit normal direction from the generator surfaces. Offset are widely used in various practical applications, such as tolerance analysis, geometric optics and robot path-planning. In some of the engineering applications, we need to extend the concept of standard offset to the generalized offset where distance offset is not necessarily constant and offset direction is not necessarily along the normal direction.

Normally, a generalized offset is functionally more complex than its progenitor because of the square root appears in the expression of the unit normal vector. For this, an approximation method of its construction is necessary.

In many situations it is necessary to fill or reconstruct certain function defined in a domain in which there is a lack of information inside one or several sub-domains (holes). In some practical cases we may have some specific geometrical constraints, of industrial or design type, for example, the case of a specified volume inside each one of these holes.

The problem of filling holes or completing a 3D surface arises in all sorts of computational graphics areas, like CAGD, CAD-CAM, Earth Sciences, computer vision in robotics, image reconstruction from satellite and radar information, etc.

In this work we present an approximation method of filling holes of the generalized offset of a surface when there is a lack of information in a sub-domain of the function that defines it. We prove the existence and uniqueness of solution, we show how to compute it and we establish a convergence result of this approximation method. Finally, we give some graphical and numerical examples.

Keywords: Generalized offset surfaces, filling holes, spline approximation, variational methods, biquadratric splines

AMS Classification: 41A15, 65D10, 65D17, 65K10

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Geometric Multilevel Methods for Isogeometric Analysis

Alvaro Pe de la Riva[1], Carmen Rodrigo[1], Francisco J. Gaspar[2]

SUMMARY

Isogeometric discretizations (IGA) are based on the use of spline-type basis functions for numerical approximations of partial differential equations (PDEs). This idea leads to the same type of parametrizations appearing in the computer aided design (CAD), that allows to capture more accurately the geometry of any computational domain. Then, IGA involves an important improvement over the classical finite element method (FEM). On the other hand, the search of an efficient solver is nowadays an important issue and multilevel methods seem to be a good choice, given that they are among the fastest solvers for FEM discretizations. However, they have to be carefully designed. For example, in multigrid methods for IGA, standard smoothers such as Gauss-Seidel do not provide a robust solver with respect to the spline degree and the convergence is seriously deteriorated. In order to find out the reason why this takes place, local Fourier analysis (LFA) is a useful tool. Moreover, it will be used for the design of adequate smoothers and then a robust solver with respect to the polynomial degree will be obtained. At this point, we propose to apply overlapping multiplicative Schwarz methods as relaxation procedures. For these geometric multilevel methods, LFA yields a prediction of a nice asymptotic convergence factor. Finally, numerical experiments will be presented to confirm the theoretical results provided by the analysis.

Keywords: Multilevel methods, Multigrid solver, Isogeometric Analysis, Local Fourier Analysis

AMS Classification: 65F10, 65M22, 65M55

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Statistical splicing of economic series by smoothing quadratic splines

L. Pedauga[1], E. Delgado–Márquez[1], M. Márquez[2], M. Pasadas[2]

SUMMARY

In this work we present a new method to solve the statistical difficult when economic series are spliced.

Therefore, we discuss the scope of some splicing tools in the literature, namely the splicing by variation and the linear interpolation methods.

These methods carry on some problems of non-linearity. In this way, certain internal inconsistency or structural incongruity can appear when the economic series are spliced. This is a serious problem for the System of National Accounts.

We introduce an approximation method for statistical splicing of economic series by smoothing quadratic splines. The proposed technique is linear and thus structurally congruent.

Finally, we show the effectiveness of our method by the results of the splicing of the GDP (Gross Domestic Product) of Venezuela between 1950 and 2012, and the corresponding economics activities.

Keywords: Splicing, economic series, quadratic spline, smoothing spline, GDP, structural congruence, Venezuela

AMS Classification: 62M10,62P20,91B82,65D07,65D10

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Reactive flows at the pore-scale of porous materials

Philippe Poncet[1,2]

SUMMARY

This talk will focus on the numerical aspects of reactive flows and pore-scale modeling, and how involving vortex-like methods can substantially improve computational efficiency, that is to say optimize the use of computing resources (computational time and memory storage). After briefly describing the numerical approach (whose hybrid aspect are introduced in [4, 3]), a few numerical simulations of calcite dissolution will be presented.

The numerical method is built specifically with a time-splitting algorithm such that velocity computing and interpolations are performed meaningfully in order to avoid useless computations (in the spirit of [5]). In practice we will show that a good strategy is to compute chemistry, diffusion of species and transport on particles, while penalization, velocity computation and viscous diffusion are performed on grids. This strategy decreases the number of interpolations thus improving the computational time. In this spirit, chemistry and hydrodynamics can be considered at different resolutions, and require only a down-scaling of solid concentration, which fits very well to GPU or hybrid CPU-GPU computing, which is currently one of our main developments [2].

Concerning reactive flows, three time scales are involved: a hydrodynamic time scale (under the second) in which the flow is able to cross the domain at the pore-scale; a time scale allowing to reach a quasi-stationary reactive state of reaction rate; a dissolution time scale for which the solid body evolves. We will consider the case of a calcite core dissolution at pH=2, in the context of a [1]. For this configuration, these three time scales are about 1s, 10s and 1h, respectively.

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Stability of domain walls in ferromagnetic rings

Romeissa Rachi[1], Gilles Carbou[1]

SUMMARY

The ferromagnetic materials are a permanent magnets that are used to store numericalsdata. On of the most innovative technologies that would ensure optimized storage with fastaccess to information is the use of stable ferromagnetic nanowires.In this work we improve the simplified ring model used by S. Labbé, Y. Privat and E.Trélat where they obtained the instability of all the wall configurations. We consider aone dimensional ferromagnetic ring model taking into account the efects of curvature andanisotropy. We describe all the planar static solutions representing domain walls and westudy their asymptotic stability. We let's show for example that the solution becomesunstable when the number of walls is great.

Keywords: Merromagnetism, ring, Landau-Lifschitz equation, stability, domain walls

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Best regularity for a Schrodinger type equation with non smooth data and interpolation spaces

J.M. Rakotoson[1]

SUMMARY

Given a vector field $U(x)$ and a nonnegative potential $V(x)$ on a smooth open bounded set Ω, we shall discuss some regularity results for the following equation

$$-\Delta\omega + U \cdot \nabla\omega + V\,\omega = f \text{ in } \Omega$$

whenever δf is a bounded Radon measure with $\delta(x)$ is the distance between x and the boundary $\partial\Omega$

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Numerical continuation of one-parameter families of periodic orbits

Andrés Riaguas[1], Eva Tresaco[2]

SUMMARY

We present here techniques to carry out the numerical continuation of one-parameter familiesof periodic orbits. The main algorithm used is due to Deprit & Henrard (1967). Thesecontinuation methods follow periodic orbits along paths in the parameter plane showing theevolution of the family and its bifurcations. This method consist of a general algorithm of computing periodic orbits, but they take ad-vantage of the simplifications due to the nature of the problems treated, namely autonomousHamiltonian systems. We will show how to combine it with other algorithms with the purposeof computing uni-parametric families of periodic orbits. The Deprit and Henrard continuation algorithm addresses a boundary value problem for the variational equations relative toa conservative dynamical system. It consists on separating the normal displacements alongan orbit from the tangential ones. This algorithm is not restricted to symmetric problems,and is valid for the computation of families of periodic orbits for variations of any parameteror integral for a conservative dynamical system with two or three degrees of freedom.

Keywords: Periodic orbits, Hamiltonian systems, bifurcations

AMS Classification: 7J45, 37M20

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Parametric inference for two imperfect repair models for gamma deteriorating systems

G. Salles, S. Mercier, L. Bordes[1]

SUMMARY

A system is considered, which is deteriorating over time according to a gamma process. The system is subject to periodic and instantaneous imperfect maintenance actions whose efficiency is measured through a parameter ρ. Each maintenance action removes a proportion ρ of the accumulated degradation either from the last maintenance action (order 1) or from the initial time (order ∞). This model is called *ARD* (*Arithmetic Reduction of Degradation*) model of order $p \in \{1, \infty\}$ and it is denoted by ARD_p. Given these models as well as the observation scheme, two classical estimation methods are considered and tested on simulated data sets in a parametric framework (Moments and Maximum Likelihood methods).

Keywords: imperfect repair models, degradation models, gamma processes

AMS Classification: 62M99, 60G51

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Mathematical challenges in computer tomography

Tomas Sauer

SUMMARY

Industrial computer tomography is significantly different from the well–known medical CT. Besides a wide variety of materials to be analyzed and significantly higher X–ray energy (up to 100 times of what is used in medical applications), the measurements are more accurate and the objects can be huge, leading to reconstructions that can be up to 1.5 TB of data. In addition, depending on the application, measurement and reconstruction may have to be extremely fast or reconstruct certain parts of the obejct with very high accuracy.

All these requirements call for new algorithms and methods in the area of 3D image processing. The talk presents some of these challenges and shows some first results and applications, for example in reconstructing cultural heritage.

Keywords: Computer Tomography, Big Data, Cultural Heritage, Wavelet Compression

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On compactness properties and ground states of an affine Laplacian

Ian Schindler

SUMMARY

We will discuss compactness properties of the affine Sobolev embedding of Gaoyong Zhang in the case $p = 2$, and existence and regularity of related minimizers.

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Galois covers of graphs and embedded topology of plane curves

Taketo Shirane[1]

SUMMARY

It is known that study of the "splitting" of plane curves with respect to Galois covers is effective to distinguish the embedded topology of certain plane curves. In this talk, we introduce the splitting graph which represents the "splitting" of plane curves. By using the splitting graph, we classify the embedded topology of Artal arrangements consisting of a smooth curve and three lines

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Hölder, Sobolev, weak-type estimates in mixed-norm with weights for parabolic equations

Pablo Raúl Stinga[1] José L. Torrea [2]

SUMMARY

We prove weighted mixed-norm $L^q_t(W^{2,p}_x)$ and $L^q_t(C^{2,\alpha}_x)$ estimates for $1 < p, q < \infty$ and $0 < \alpha < 1$ and weighted mixed weak-type estimates for q = 1, as well as a.e. pointwise formulas for derivatives, for solutions $u = u(t; x)$ to parabolic equations of the form

$$\partial_t u - a^{ij}(t)\partial_{ij} u + u = f, \quad t \in R^n, x \in R^n$$

and for the Cauchy problem

$$\partial_t v - a^{ij}(t)\partial_{ij} v + v = f, \quad \text{for } t > 0; x \in R^n$$

$$v(0, x) = g \quad \text{for } x \in R^n$$

The coefficients $a(t) = (a^{ij}(t))$ are just bounded, measurable, symmetric and uniformly elliptic. Furthermore, we show strong, weak type and BMO estimates with parabolic Muckenhoupt weights. It is quite remarkable that most of our results are new even for the classical heat equation

$$\partial_t u - \Lambda u + u = f.$$

Keywords: Heat equation, weighted Sobolev estimate, mixed-norm estimate, semigroups.

AMS Classification: Primary: 35K10, 35B45, 42B37; Secondary: 58J35, 42B20

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A parabolic problem with constraint in population dynamics

Yassine TAHRAOUI

SUMMARY

Variational inequalities are well known in the literature of applied mathematics and lead to many applications. They are related to obstacle problems and free boundary problems. In this talk we present a parabolic problem with a unilateral constraint: the existence of solution and Lewy-Stampacchia's inequality; in the context of a family of operators more general than the monotonous case studied in [1]. This applies for example to a population forced to live in a particular area of space [2].

Keywords: Variational inequalities, pseudo-monotone operator, Lewy-Stampacchia's inequality.

AMS Classification: 35K86, 35R35

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The origin of the p-Laplacian and A. Missbach

Peter Takáč[1], Jiří Benedikt[2], Petr Girg[2], Lukáš Kotrla[2]

SUMMARY

We describe the historical process of derivation of the p-Laplace operator from a nonlinear Darcy law and the continuity equation. The story begins with nonlinear flows in channels and ditches. As the nonlinear Darcy law we use the power law discovered by O. Smreker and verified in experiments by A. Missbach for flows through porous media in one space dimension. These results were generalized by S. A. Christianovitch and L. S. Leibenson to porous media in higher space dimensions. We provide a brief description of Missbach's experiments.

Keywords: porous medium, filtration, nonlinear Darcy law, pressure-to-velocity power law, p-Laplacian

AMS Classification: 76S05, 35Q35, 35–03 Historical

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On defect of compactness for Sobolev embeddings

Cyril Tintarev

SUMMARY

Defect of compactness for an continuous embedding of a Banach space X into a metric space Y is the difference between a weakly convergent sequence $u_k \in X$ and its weak limit u, taken modulo sequences vanishing in Y . For many pairs of spaces one knows a group G of isometries on X that allows to represent defect of compactness as profile decomposition - a sum of mutually decoupled terms of the form $g_k w$ with $g_k \in G$. Element $w \in X$ is given as the weak limit of $g_k^{-1} u_k$ and is called a concentration profile. Existence of profile decomposition in presence of a group has been proved for general Banach spaces. Profile decompositions are applied in PDE to prove convergence of putative approximate solutions when one has no compact embedding at hand. In such settings concentration profiles satisfy some equation at infinity and there may be reasons, e.g. Liouville theorems, for them to be zero, resulting in zero defect of compactness and thus convergence. In this talk we outline several recent results concerning Sobolev spaces on manifolds, where profile decomposition is constructed without a group of isometries and some applications to semilinear elliptic problems on Riemannian manifolds. The main part of this work is done jointly with Leszek Skrzypczak.

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Topology of plane curves and "arithmetic" of double covers of $\mathbb{P}^2$

Hiroo Tokunaga[1]

SUMMARY

Let C be a reduced plane algebraic curve $\subset \mathbb{P}^2$. The combinatorics of C (or The combinatorial type of C means that roughly speaking,

Data on

- the degrees of irreducible components,
- how irreducible components intersects,
- the topological types of singularities and so on.

One of naive questions is:

Problem: What can we say about the topology of $(\mathbb{P}^2, C)$ just from the combinatorics of C?

Since the topology of $(\mathbb{P}^2, C)$ is not necessarily determined by the combinatorics of C, the above question is subtle. To consider the above problem, various topological invariants have been used. In this talk, we introduce a new point of view "arithmetic" of double covers and explain how it works through some examples.

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A domain decomposition strategy for a very high-order finite volumes scheme applied to cardiac electrophysiology

R. Turpault Y. Coudière[2]

SUMMARY

The propagation of an electrical action potential in the heart may be modelled by the monodomain model:

$$\partial_t V + I_{ion}(V,w) = \div D\nabla V, (1), \qquad \partial_t w = G(V,w), (2)$$

where V is the transmembrane voltage, $I_{ion}(V,w)$ is the normalized ionic current, and D is the normalized diffusion tensor. Equation (2) is the system of m nonlinear ordinary differential equations called the ionic model, and specified through the function $G(V,w)$. Let us mention that realistic ionic models are stiff and contain some variables which lie into a domain of admissibility (e.g. ionic concentrations). Moreover, this model generates the propagation of very stiff depolarization fronts. These difficulties and the fact that simulations are required on long periods of time make the design of efficient numerical schemes critical. We will introduce the scheme proposed in [1], which approximates the solutions of (1)-(2). It is a very high-order scheme (up to order 6) based on a MOOD paradigm with two polynomial reconstructions: one on the cells et the other on the interfaces (see references therein and also [2]). Furthermore, in order to be able to perform realistic computation, we propose a domain decomposition method: the computational domain is divided into subdomains and the reconstruction stencils are adapted to theses subdomains. In particular, they are forced to stay either in the subdomain or in the immediate vicinity of it (typically in the first layer of cells). In the presentation, we will detail the procedure and the different validation test-cases which show the technique's efficiency.

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Fifteenth International Conference Zaragoza-Pau on Mathematics and its Applications
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Periodic solutions for impulsive differential equations

José Manuel Uzal[1]

SUMMARY

The main objective of this talk is to present some results on the existence of periodic solutions for some impulsive differential equations problems.

Many evolution processes are characterized by the fact that they experience a sudden change in their state at certain moments of time. These changes can be assumed to occur instantaneously[1]. Mathematical models in aircraft control, population dynamics or in economy show impulsive effects [2].

Two different problems will be considered. Firstly, a first-order differential equations with the possible presence of singularities and impulses is studied. The impulses are assumed to happen on the position and at instants of time fixed beforehand. Under some easy-to-check hypotheses, the existence of positive and periodic solutions will be proved and some examples will be presented. Secondly, a second-order differential equations is considered with state-dependent impulses at both the position and its derivative. This means that the instants of impulsive effects depend on the solutions and they are not fixed beforehand, making the study of this problem more difficult. Once again, the existence of periodic solutions will be proved.

Keywords: impulsive differential equations, periodic solutions

AMS Classification: 34B37, 34A37

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On a nonlocal Stochastic PDE

Guy Vallet[1], Ujjwal Koley[2]

SUMMARY

Let $\big(\Omega, \mathbb{P}, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}\big)$ be a filtered probability space satisfying the usual hypothesis i.e., $\{\mathcal{F}_t\}_{t\geq 0}$ is a right-continuous filtration such that $\mathcal{F}_0$ contains all the $\mathbb{P}$-null subsets of $(\Omega, \mathcal{F})$. We are interested in the existence an uniqueness of an entropy $L^2(\mathbb{R}^d)$-valued predictable process $u(t,\cdot)$ which satisfies the Cauchy problem

$$du + \mathcal{L}_\lambda \phi(u) - \operatorname{div} \vec{f}(u)\, dt = h(u)\, dW(t) \quad \text{in } Q, \qquad u(t=0) = u_0 \quad \text{in } \mathbb{R}^d,$$

where $Q = \mathbb{R}^d \times (0,T)$ with $T > 0$ fixed, $u_0(x)$ is the given initial function, $\mathcal{L}_\lambda$ is the λ-fractional Laplace operator, $\vec{f} : \mathbb{R} \mapsto \mathbb{R}^d$, $\phi, h : \mathbb{R} \mapsto \mathbb{R}$ are a given (sufficiently smooth) functions and $W(t)$ is a real valued Brownian noise.

Keywords: Stochastic problems, entropy solution, fractional operators

AMS Classification: 60H15, 35R11, 35L65

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Well-posedness for a class of nonlinear SPDEs with strongly continuous perturbation

Aleksandra Zimmermann

SUMMARY

We consider the stochastic evolution equation

$$du - \nabla(a(x, u, Du) + F(u))dt = \Phi\, dW$$

for $T > 0$, on a bounded Lipschitz domain D with homogeneous Dirichlet boundary conditions and initial condition in $L^2(D)$. The main technical difficulties arise from the nonlinear diffusion-convection operator which is defined by a Carathéodory function $a = a(x, \lambda, \xi)$ satisfying appropriate growth and coercivity assumptions and $F : \mathbb{R} \to \mathbb{R}^d$ Lipschitz continuous. On the right-hand side, we consider an additive stochastic perturbation with respect to a cylindrical Wiener process with values in $L^2(D)$. We obtain approximate solutions by a semi-implicit time discretization. Adjusting the method of stochastic compactness to our setting, we are able to pass to the limit in the approximate equation. We show an L^1-contraction principle and obtain existence and uniqueness of (stochastically) strong solutions.

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