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On a class on doubly nonlinear parabolic equations

S.Antontsev[1], **S.Shmarev**[2]

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SUMMARY

We present recent results on the existence and localization properties of solutions of parabolic equations with double variable nonlinearity:

$$\partial_t \left(|u|^{m(x,t)} \operatorname{sign} u \right) - \sum_{i=1}^{n} D_i \left(|D_i u|^{p_i(x,t)-2} D_i u \right) + f(x,t,u) = 0$$

where (x,t) denotes the points of the cylinder $Q = \Omega \times (0,T)$, $p_i(x,t) > 1$ and $m(x,t) > 0$ are given functions. The function $f(x,t,u)$ models the presence of absorption or reaction. The following issues are discussed:

- sufficient conditions for the existence of strong solutions,
- energy estimates for strong solutions,
- sufficient conditions of the finite time vanishing ($f < 0$),
- nonexistence of global in time solutions (blow-up, $f > 0$),
- the possibility of extinction of solutions in a finite time in the limiting cases when $f \equiv 0$ and $p_i(x,t) \to 2$, $m(x,t) \to 1$ as $t \to \infty$, and the equation eventually becomes linear.

The presentation mostly follows the results of [1]-[3].

Keywords: Parabolic equation, double variable nonlinearity, blow-up, extinction, decay rates

AMS Classification: 35K55, 35K65, 35K67

References

[1] S. N. Antontsev and S. I. Shmarev. Anisotropic parabolic equations with variable nonlinearity. Publ. Mat. 53 (2009), no.2, pp. 355–399.

[2] S. N. Antontsev and S. I. Shmarev. Doubly degenerate parabolic equations with variable nonlinearity I: Existence of bounded strong solutions. Adv. Differential Equations 17 (2012), no. 11-12, 1181–1212.

[3] S. N. Antontsev and S. I. Shmarev. Doubly degenerate parabolic equations with variable nonlinearity II: Blow-up and extinction in a finite time. Nonlinear Anal. 95 (2014), 483–498.

[1]CMAF
University of Lisbon, Portugal
antontsevsn@mail.ru

[2]Mathematics Department
University of Oviedo, Spain
shmarev@uniovi.es