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Nonconforming finite element approximation for an elliptic interface problem with NXFEM method

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SUMMARY

Several finite element methods have been proposed in the last years in order to treat cut meshes. One of them is NXFEM, introduced by A. Hansbo and P. Hansbo in [3] and based on the use of Nitsche's method to treat the transmission conditions on the interface.
Our goal is to extend NXFEM to the case of nonconforming finite elements in order to apply it, in the mid-term, to the coupling of Newtonian and non-Newtonian fluids. The application that we have in mind is the modelling of red blood cells in a blood flow. So far, we have considered the Darcy and Stokes equations.
In this talk, we propose two approaches to tackle this problem. We mainly focus on elliptic equations with discontinuous coefficients. We consider triangular meshes and P_1 nonconforming elements of Crouzeix-Raviart [2]. The first approach consists in employing the classical Crouzeix-Raviart elements and in adding some stabilisation terms on each part of the edges cut by the interface. In this case, the additional stabilisation terms compensate the nonconformity error on the cut edges. In the second one, we modify the basis functions on the cut triangles such that the new degrees of freedom are associated to only one subdomain. The difficulty now lies in estimating the interpolation error in a robust way.

Both approaches yield well-posedness of the discrete problem and optimal a priori error estimates. Moreover, we have shown that the first method converges towards the second one when the (numerical) stabilisation parameter tends towards infinity. Numerical tests will be presented in order to validate the previous theoretical results.

Keywords: NXFEM, nonconforming finite elements, a priori error estimate, robustness

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