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A new method for the approximation of second-order differential equations with a large parameter

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SUMMARY

We consider the linear second order differential equation

$$y'' = \left[\frac{\Lambda^2}{x^\alpha} + g(x)\right] y,$$

where $x \in [0, X]$, $X > 0$, $\alpha \in (-\infty, 2)$, Λ is a large complex parameter and g is a continuous function. The asymptotic method designed by F. Olver [Olver, 1974] gives the Poincaré-type asymptotic expansion of two independent solutions of the equation in inverse powers of Λ. We add initial conditions to the differential equation and consider the corresponding initial value problem. By using the Green's function of an auxiliary problem, we transform the initial value problem into a Volterra integral equation of the second kind. Then using a fixed point theorem, we construct a sequence of functions that converges to the unique solution of the problem. This sequence has also the property of being an asymptotic expansion for large Λ of the solution of the problem. Moreover, we show that the idea may be applied also to nonlinear differential equations with a large parameter.

Keywords: Second order differential equations. Turning points. Singular points. Volterra integral equations of the second kind. Asymptotic expansions. Green functions. Fixed point theorems. Airy functions. Bessel functions.

AMS Classification: 34A12, 45D05, 41A58, 41A60, 34B27

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