Thirteenth International Conference Zaragoza-Pau on Mathematics and its Applications
Jaca, September 15–18th 2014

# Kernels of convolution and subdivision operators

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## SUMMARY

As indicated by the name, *convolution operators* act on discrete data, defined on $\mathbb{Z}^s$ by means of convolution

$$c \mapsto f * c := \sum_{\alpha \in \mathbb{Z}^s} a(\cdot - \alpha) c(\alpha)$$

with an *impulse response* $f$ which is supposed to be a finitely supported function on $\mathbb{Z}^2$ as well. A natural question to ask is: what are the kernels of such operators, or, equivalently, the homogeneous solutions of the associated (partial) difference equation. In the univariate case this is well–known and the kernel spaces are exponential polynomial spaces where the frequencies of the exponentials are the locations, the degrees of the polynomials the multiplicities of the zero of the *symbol*

$$f^*(z) := \sum_{\alpha \in \mathbb{Z}^s} f(\alpha) z^\alpha, \qquad z \in (\mathbb{C} \setminus \{0\})^s .$$

In several variables, things become more interesting as the multiplicity theory of common zeros of polynomials is not a matter of of counting any. Surprisingly, the original theory of multiplicities in [1, 2] is fairly old and has originally been developed to solve the problem of determining the kernels of partial differential operators with constant coefficients. It turns out that a similar theory also holds for difference operators which can also be applied to *subdivision operators* where such annihilators are a fundamental tool in any sort of convergence analysis.


**Keywords:** convolution, subdivision, exponential polynomials

**AMS Classification:** 39A14, 65D17, 65Q10

## References

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