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On the quantile functions of the Log–Lindley distribution and a discrete Lindley distribution†

P. Jodrá[1]

SUMMARY

The Lindley distribution was introduced by the British statistician Dennis V. Lindley (1923–2013) in the context of Bayesian Statistics. Jodrá [3] shown that this probability distribution has the variate generation property (vgp), that is, the quantile function (qf) can be given in closed form. The qf of a random variable T is defined by $Q_T(u) := \inf\{t \in \mathbb{R} : \; F_T(t) \geq u\}$, $0 < u < 1$, where $F_T(t) := P(T \leq t)$. The vgp is a remarkable property since it implies that random samples can be computer-generated by means of the inverse transform method.

Using the Lindley distribution, Gómez-Déniz et al. [2] have derived the Log–Lindley distribution, which has been proposed as an alternative to the beta distribution, and Gómez-Déniz and Calderín-Ojeda [1] have introduced a discrete Lindley distribution by discretizing the Lindley distribution, which has been proposed as an alternative to the Poisson model.

In this work, we show that the Log–Lindley distribution (X) and the aforementioned discrete Lindley distribution (Y) have the vgp. More specifically, their qfs can be expressed in closed form in terms of the Lambert W function as follows

$$Q_X(u; \lambda, \sigma) = \exp\left\{\frac{1+\lambda\sigma}{\sigma}\right\} \exp\left\{\frac{1}{\sigma} W_{-1}\left(-\frac{u(1+\lambda\sigma)}{\exp\{1+\lambda\sigma\}}\right)\right\}, \quad 0 < u < 1, \quad \lambda \geq 0, \sigma > 0,$$

$$Q_Y(u; \lambda) = \left\lceil -2 + \frac{1}{\log\lambda} + \frac{1}{\log\lambda} W_{-1}\left((u-1)(1-\log\lambda)\lambda e^{-1}\right)\right\rceil, \quad 0 < u < 1, \quad 0 < \lambda < 1,$$

where W_{-1} denotes the negative branch of the Lambert W function and $\lceil\cdot\rceil$ stands for the ceiling of a real number. The vgp reinforces the interest in these models as alternatives to the beta and Poisson distributions since these classical distributions do not have that property.

Keywords: Lindley distribution, Lambert W function, simulation

AMS Classification: 65C05, 33B30

References

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[1]Dpto. de Métodos Estadísticos, Universidad de Zaragoza
C/ María de Luna 3, 50018 Zaragoza, Spain
E-mail: pjodra@unizar.es

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