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Nonlinear and singular parabolic equation

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SUMMARY

We present in this talk some existence results of positive weak solution for the following singular non-linear problem with homogeneous Dirichlet boundary conditions:

$$\begin{cases} \partial_t u - \Delta_p u = u^{-\delta} + f(x,u,\nabla u) \ \text{ in } (0,T)\times\Omega = Q_T, \\ u = 0 \ \text{ on } (0,T)\times\partial\Omega, \ u > 0 \text{ in } Q_T, \\ u(0,x) = u_0 \geq 0 \text{ in } \Omega, \end{cases}$$

where Ω stands for a regular bounded domain of $\mathbb{R}^N$, $\Delta_p u$ is the p-Laplacian defined by $\Delta_p u = div\left(|\nabla u|^{p-2}|\nabla u|\right)$, $2 \leq p < \infty$, $\delta > 0$ and $T > 0$. The nonlinear term $f : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ is a Carathéodory function satisfying the growth condition

$$f(x,s,\xi) \leq \left(as^{q-1} + b\right) + c|\xi|^{p-\frac{p}{q}} \text{ for a.a. } x \in \Omega, \ s \in \mathbb{R}_+ \text{ and } |\xi| \geq M$$

where $a, c, M > 0$ and $b \geq 0$ are some constants and $q \in [p, p^*)$ where $p^* = \frac{pN}{N-p}$ if $p < N$ and $p^* = \infty$ if $p \geq N$. We prove for any nonnegative initial data $u_0 \in L^r(\Omega)$ with $r \geq 2$ large enough and for any $\delta > 0$, the existence of at least one weak solution to (P). In the case $\delta < 2 + \frac{1}{p-1}$, we prove the uniqueness of the solution and further regularity results. For that we use some estimates based on logarithmic Sobolev inequalities to get ultracontractivity of the associated semi-group. This approach was already used for quasilinear parabolic equations by P. Takáč [3]. We use also the local regularity result of E. DiBenedetto (see [2] which establish a Hölder continuity of nonnegative local weak solutions of parabolic equations).

Keywords: p-Laplacian operator, comparaison principle, singular problems, parabolic problems, a priori estimates.

References

[1] M. Badra, K. Bal, J. Giacomoni. Some results about a quasilinear singular parabolic equation. *Differ. Equ. Appl.* **3**, 609–627, 2011

[2] E. DiBenedetto, A. Freidman, Hölder estimates for non-linear degenerate parabolic equations, J. Reine Angew. Math. 357 (1985), 1–22.

[3] P. Takáč. L^∞-Bounds for weak solutions of an evolutionary equation with the p-Laplacian. In *Proceeding of the conference Held in Milovy, Bohemian-Moravian Uplands, May 28-June 2 2004*, Pavel Drábek, Jiří Rákosník, pp. 327–354. 2004.

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